Structural Wood Design Using ASD and LRFD

By John "Buddy" Showalter, P.E.

Introduction

A new publication entitled *Structural Wood Design Using ASD and LRFD* is being developed as a companion design tool to the *2005 National Design Specification® (NDS®) for Wood Construction*. It will be available beginning in the Spring of 2005 through the American Forest & Paper Association (AF&PA). The authors are Dr. Dan L. Wheat, P.E. of the University of Texas at Austin and Dr. Steven M. Cramer, P.E. of the University of Wisconsin–Madison. The book is intended to aid instruction on structural design of wood structures using both allowable stress design (ASD) and load and resistance factor design (LRFD). It will allow direct comparison of ASD and LRFD for wood structures on a problem by problem basis.

Background

Serving as the basis for the new document, a design aid, entitled *LRFD Solved Example Problems for Wood Structures*, which was published in 2000 as a companion to AF&PA's *LRFD Manual*, has been updated to include parallel ASD solutions to the 40 LRFD example problems previously developed. It was originally co-sponsored by the American Wood Council of AF&PA, Southern Pine Council (Southern Forest Products Association and the Southeastern Lumber Manufacturers Association) and the Wood Truss Council of America, and published by the International Codes Council.

The design examples in *Structural Wood Design Using ASD and LRFD* range from simple to complex and cover many design scenarios. This design aid is intended for use by practicing engineers, many of whom currently use ASD, but who may want to compare and contrast it with LRFD; and by academics, whose teaching objectives may vary. Some problems have been posed as stand-alone problems, but others belong to a set in which all examples are associated with one structure. The focus of this design tool is education; it is not to propose specific designs or details. It is hoped that the format, which has facing pages with ASD and LRFD solutions, will allow readers to verify the comparable work efforts and common design equations of ASD and LRFD. As well, the new design aid will include examples that incorporate such provisions as those for local stresses in fastener groups, namely row tear-out and group tear-out, which were not a part of AF&PA's *LRFD Manual*.

Example Problem

To illustrate the content of *Structural Wood Design Using ASD and LRFD*, an example problem has been reproduced on the following pages. It is a uniformly loaded beam with an overhang, and it is to be designed for the simultaneous application of dead and snow load. All problems will be in this general format; that is, the ASD solution is shown on the left page, with the LRFD solution shown on the facing page. This will allow for an easy comparison of the two sets of methods.

Solutions have been developed using Mathcad® Professional software by MathSoft®, Inc. Those unfamiliar with Mathcad should note that the symbol ":=" is equivalent to the traditional equals "=" sign.

Problem 10. Cantilevered Glulam Beam Design

Select the size of the glued laminated timber beam to resist an unfactored uniform dead load of 20 lb per foot (includes beam self-weight) and 180 lb per foot of snow load. Full lateral support is provided only at the pin and roller supports. Normal temperature and dry conditions prevail. Use visually graded Southern Pine combination 20F-V5. Only flexure and shear are to be checked.

ASD Solution Problem 10

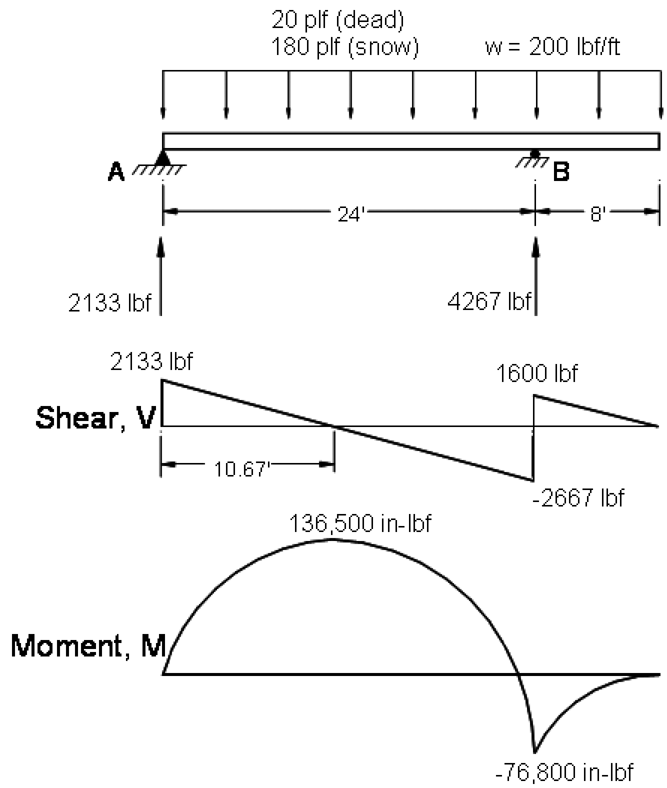
Structural Analysis

$$P_{dead} := 20\,\frac{lbf}{ft}$$

$$P_{snow} := 180\,\frac{lbf}{ft}$$

$$w := P_{dead} + P_{snow}$$

$$w = 200\,\frac{lbf}{ft}$$



Equations of Equilibrium

$$Reaction_B := w \cdot 32\,ft \cdot 16\,\frac{ft}{24\,ft} \qquad Reaction_B = 4.267 \times 10^3\ lbf$$

$$Reaction_A := w \cdot 32\,ft - Reaction_B \qquad Reaction_A = 2.133 \times 10^3\ lbf$$

$$V_A := Reaction_A$$

$$V_{B\_left} := V_A - w \cdot 24\,ft \qquad V_{B\_left} = -2.667 \times 10^3\ lbf$$

$$V_{B\_right} := Reaction_B + V_{B\_left} \qquad V_{B\_right} = 1.600 \times 10^3\ lbf$$

$$M_{positive} := V_A \cdot \left(\frac{V_A}{w}\right) \cdot \frac{1}{2} \qquad M_{positive} = 1.365 \times 10^5\ lbf \cdot in$$

$$M_{negative} := w \cdot 8\,ft \cdot 4\,ft \qquad M_{negative} = 7.680 \times 10^4\ lbf \cdot in$$

LRFD Solution Problem 10

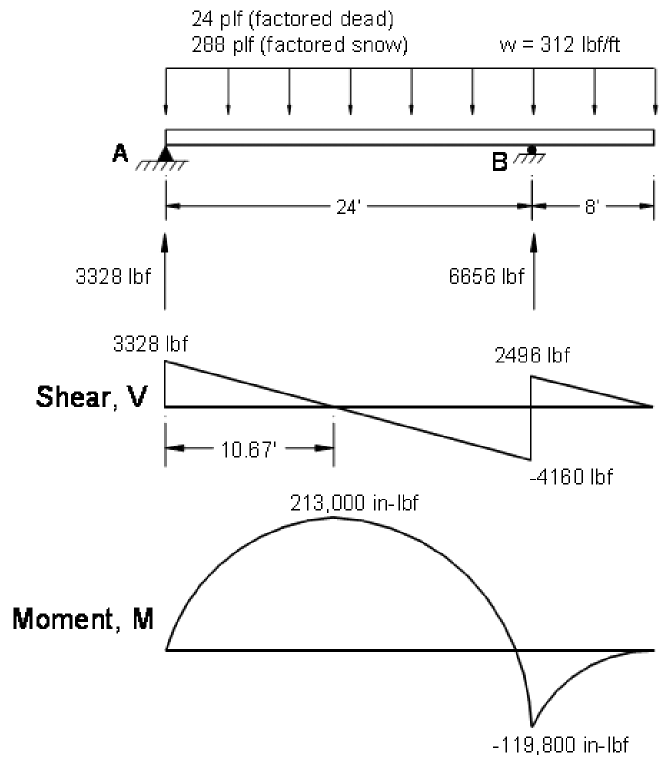
Structural Analysis

$P_{dead} := 20\,\frac{lbf}{ft}$

$P_{snow} := 180\,\frac{lbf}{ft}$

$w := 1.2P_{dead} + 1.6P_{snow}$

$w = 312\,\frac{lbf}{ft}$



Equations of Equilibrium

$Reaction_B := w \cdot 32ft \cdot 16\frac{ft}{24ft}$ $\qquad Reaction_B = 6.656 \times 10^3\ lbf$

$Reaction_A := w \cdot 32ft - Reaction_B$ $\qquad Reaction_A = 3.328 \times 10^3\ lbf$

$V_A := Reaction_A$

$V_{B\_left} := VA - w \cdot 24ft$ $\qquad V_{B\_left} = -4.160 \times 10^3\ lbf$

$V_{B\_right} := Reaction_B + V_{B\_left}$ $\qquad V_{B\_right} = 2.496 \times 10^3\ lbf$

$M_{positive} := V_A \cdot \left(\frac{V_A}{w}\right) \cdot \frac{1}{2}$ $\qquad M_{positive} = 2.130 \times 10^5\ lbf \cdot in$

$M_{negative} := w \cdot 8ft \cdot 4ft$ $\qquad M_{negative} = 1.198 \times 10^5\ lbf \cdot in$

ASD–Problem 10

Member Information

Reference Design Values
NDS Supplement Table 5A

$F_{b\_tension} := 2000\ psi$

$F_{b\_compression} := 2000\ psi$

$F_v := 300\ psi$

$E_y := 1400000\ psi$

$E_{y\ min} := 730000\ psi$

Section Properties

$Length := 24 \cdot 12\ in$ NDS 3.3.3.4

$Depth := \left(9 + \frac{5}{8}\right) in$ Trial size

$Width := 5\ in$ Trial size

$Area := Depth \cdot Width$

$Area = 48.125\ in^2$

$$S_{xx} := \frac{\left[\frac{(Width \cdot Depth^3)}{12}\right]}{\left(\frac{Depth}{2}\right)}$$

$S_{xx} = 77.201\ in^3$

Adjustment Factors
NDS Table 5.3.1

$C_D := 1.15$	Load duration factor
$C_M := 1.0$	Wet service factor
$C_t := 1.0$	Temperature factor
$C_{fu} := 1.0$	Flat use factor
$C_c := 1.0$	Curvature factor

Design Calculations
Preliminary Design

The preliminary sizing of members is done in a variety of ways by designers. In general, a beam must be examined for several load combinations and the designer is responsible for determining the critical or controlling one. This can be done by selecting which load combination gives the highest M/C_D value [not the highest moment alone]. Once the critical load combination is determined, an estimated section modulus could be calculated as $\frac{M}{C_D F_b}$, leaving off for simplicity other adjustment factors of F_b. However, there are at least four other options for getting trial member sizes. One is simply to guess a beam size. Another is to calculate M/F_b. Yet another is to select a section modulus to satisfy deflection criteria–but none is specified in this problem–which often controls. The last option is to calculate the required section modulus using the adjusted design value, F'_b, that is, with all of the adjustment factors applied to F_b, but realizing that some adjustment factors, such as C_V,

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Member Information

Reference Design Values
NDS Supplement Table 5A

The following reference design values are tabulated in the *NDS Supplement*. They will be adjusted for LRFD later in the solution when other wood specific adjustments are applied

$F_{b\_tension} := 2000\ psi$

$F_{b\_compression} := 2000\ psi$

$F_v := 300\ psi$

$E_y := 1400000\ psi$

$E_{y\ min} := 730000\ psi$

Section Properties

$Length := 24 \cdot 12\ in$ NDS 3.3.4.4

$Depth := \left(9 + \frac{5}{8}\right) in$ Trial size

$Width := 5\ in$ Trial size

$Area := Depth \cdot Width$

$Area = 48.125\ in^2$

$$S_{xx} := \frac{\left[\frac{(Width \cdot Depth^3)}{12}\right]}{\left(\frac{Depth}{2}\right)}$$

$S_{xx} = 77.201\ in^3$

Adjustment Factors
NDS Table 5.3.1

$C_M := 1.0$	Wet service factor
$C_t := 1.0$	Temperature factor
$C_{fu} := 1.0$	Flat use factor
$C_c := 1.0$	Curvature factor
$\lambda := 0.8$	Time effect factor
$\phi_b := 0.85$	Bending resistance factor
$\phi_v := 0.75$	Shear resistance factor
$\phi_s := 0.85$	Stability resistance factor

$K_{F\_b} := \frac{2.16}{\phi_b}$	Format conversion factor for bending
$K_{F\_s} := \frac{1.5}{\phi_s}$	Format conversion factor for stability
$K_{F\_s} := \frac{2.16}{\phi_v}$	Format conversion factor for shear

The K_F factors convert reference design values and moduli (for stability) to LRFD reference resistances –see Table N1 Appendix N

Design Calculations
Preliminary Design

The preliminary sizing of members is done in a variety of ways by designers. In general, a beam must be examined for several load combinations and the designer is responsible for determining the critical or controlling one. This is easily done by selecting which load combination gives the highest M/λ value [not the highest moment alone]. Once the critical load combination is determined, an estimated section modulus could be calculated as

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Preliminary Design ***(continued from page 14)***

may have to be revisited if the beam size later changes. A similar process applies if shear is used to determine a trial section. There is no one right way to do this and the choice of method often depends on the experience of the designer. A trial section of 5 in. by 9-5/8 in. was chosen to start the solution.

Adjustment Factor Calculations	Comments
Note, C_V and C_L are not used in the same calculation for moment capacity. The lower of the two is used.	
Volume Factor	
$C_V := \left(\frac{21}{21.34}\right)^{\frac{1}{20}} \cdot \left(\frac{12\,\text{in}}{\text{Depth}}\right)^{\frac{1}{20}} \cdot \left(\frac{5.125\,\text{in}}{\text{Width}}\right)^{\frac{1}{20}}$	NDS 5.3.6
	Eq. 4.1: 21.34 ft is the distance between points of zero moment
$C_V = 1.012$	This value cannot exceed 1, therefore $C_V = 1$.
Beam Stability Factor	
$L_u := \text{Length}$	NDS 3.3.3
$\frac{L_u}{\text{Depth}} := 29.922$	The unsupported length is taken as the span between the supports
$l_e := 1.63 \cdot L_u + 3 \cdot \text{Depth}$	Effective length chosen from Table 3.3.3
$l_e = 498.3\text{ in}$	
$R_B := \sqrt{\frac{(l_e \cdot \text{Depth})}{\text{Width}^2}}$	Eq. 3.3-5
$R_B = 13.851$	R_B is less than 50 as required
$COV_E := 0.10$	Appendix F, Table F1
$E'_{min} := E_{y\,min} \cdot C_M \cdot C_t$	
$E'_{min} := 7.3 \times 10^5\text{ psi}$	
$F_{b\_star} := F_{b\_tension} \cdot C_D \cdot C_M \cdot C_t \cdot C_{fu} \cdot C_c$	F_{b_star} is the same as F^*_b in NDS 3.3.3
$F_{b\_star} = 2.300 \times 10^3\text{ psi}$	All factors except for C_L or C_V multiplied by the NDS reference design value.
$F_{bE} := 1.20 \cdot \frac{E'_{min}}{{R_B}^2}$	
$\alpha := \frac{F_{bE}}{F_{b\_star}}$	
$\alpha = 1.985$	
$C_L := \left(\frac{1+\alpha}{1.9}\right) - \sqrt{\left(\frac{1+\alpha}{1.9}\right)^2 - \frac{\alpha}{0.95}}$	Eq 3.3-6
$C_L = 0.956$	C_L controls, rather than C_V

Preliminary Design ***(continued from page 15)***

$\frac{M}{\lambda K_F \phi_b F_b}$, leaving off for simplicity other adjustment factors of F_b. Note that this estimate includes the conversion–by means of K_F–of a normal duration allowable design value F_b to a short-term reference resistance, which then is adjusted by λ and ϕ. However, there are at least four other options for getting section modulus. One is simply to guess a beam size. Another is to calculate M/F_b. Yet another is to select a section modulus to satisfy deflection criteria–but none is specified in this problem–which often controls. The last option is to calculate the required section modulus using the adjusted design value, F'_b, that is, with all of the adjustment factors applied to F_b, but realizing that some adjustment factors, such as C_V, may have to be revisited if the beam size later changes. A similar process applies if shear is used to determine a trial section. There is no one right way to do this and the choice of method often depends on the experience of the designer. A trial section of 5 in. by 9-5/8 in. was chosen to start the solution.

Adjustment Factor Calculations	**Comments**
Note, C_V and C_L are not used in the same calculation for moment capacity. The lower of the two is used.	
Volume Factor	
$C_V := \left(\frac{21}{21.34}\right)^{\frac{1}{20}} \cdot \left(\frac{12\,in}{Depth}\right)^{\frac{1}{20}} \cdot \left(\frac{5.125\,in}{Width}\right)^{\frac{1}{20}}$	NDS 5.3.6 Eq. 4.1: 21.34 ft is the distance between points of zero moment
$C_V = 1.012$	This value cannot exceed 1, therefore $C_V = 1$.
Beam Stability Factor	
$L_u :=$ Length	NDS 3.3.3
$\frac{L_u}{Depth} := 29.922$	The unsupported length is taken as the span between the supports
$l_e := 1.63 \cdot L_u + 3 \cdot Depth$	Effective length chosen from Table 3.3.3
$l_e = 498.3$ in	
$R_B := \sqrt{\frac{(l_e \cdot Depth)}{Width^2}}$	Eq. 3.3-5
$R_B = 13.851$	R_B is less than 50 as required
$COV_E := 0.10$	Appendix F, Table F1
$E'_{min} := \phi_s \cdot K_{F\_s} \cdot E_{y\,min} \cdot C_M \cdot C_t$	
$E'_{min} := 1.095 \times 10^6$ psi	
$F_{b\_star} := \lambda \cdot \phi_b \cdot K_{F\_b} \cdot F_{b\_tension} \cdot C_M \cdot C_t \cdot C_{fu} \cdot C_c$	
$F_{b\_star} = 3.456 \times 10^3$ psi	F_{b_star} is the same as F^*_b in NDS 3.3.3
$F_{bE} := 1.20 \cdot \frac{E'_{min}}{R_B^2}$	All factors except for C_L or C_V multiplied by the NDS design value.
$\alpha := \frac{F_{bE}}{F_{b\_star}}$	
$\alpha = 1.982$	
$C_L := \left(\frac{1+\alpha}{1.9}\right) - \sqrt{\left(\frac{1+\alpha}{1.9}\right)^2 - \frac{\alpha}{0.95}}$	Eq 3.3-6
$C_L = 0.956$	C_L controls, rather than C_V

Solve for Adjusted Stress	ASD-Problem 10
Bending Design $F'_b := F_{b\_tension} \cdot C_L \cdot C_D \cdot C_M \cdot C_t \cdot C_{fu} \cdot C_c$ $F'_b := 2198\ \text{psi}$ $M_{positive} = 136533\ \text{lbf} \cdot \text{in}$ $f_b := \frac{M_{positive}}{S_{xx}}$ $f_b := 1769\ \text{psi}$ $f_b < F'_b$	NDS 3.3
Check Shear $F'_v := F_v \cdot C_D \cdot C_M \cdot C_t$ $F'_v = 345\ \text{psi}$ $V_{B\_left} = -2.667 \times 10^3\ \text{lbf}$ $f_v := \frac{3 \cdot \lvert V_{B\_left} \rvert}{2 \cdot \text{Area}}$ $f_v = 83.1\ \text{psi}$ $f_v < F'_v$	NDS 3.4 The shear is checked just to the left of the roller support at the point of maximum shear force. Shear checks in this example, but if it did not, the provisions of NDS 3.4.3.1 could be invoked. This section allows the shear to be calculated at a distance equal to the depth of the beam from the support.
Use 5-inch by 9-5/8-in 20F-V5 glulam	

	LRFD–Problem 10
Solve for Adjusted Stress	
Bending Design $F'_b := \lambda \cdot \phi_b \cdot K_{F\_b} \cdot F_{b\_tension} \cdot C_L \cdot C_M \cdot C_t \cdot C_{fu} \cdot C_c$ $F'_b := 3302$ psi $M_{positive} = 212992$ lbf·in $f_b := \frac{M_{positive}}{S_{xx}}$ $f_b := 2759$ psi $f_b < F'_b$	NDS 3.3
Check Shear $F'_v := \lambda \cdot \phi_v \cdot K_{F\_v} \cdot F_v \cdot C_M \cdot C_t$ $V_{B\_left} = -4.16 \times 10^3$ lbf $f_v := \frac{3 \cdot \lvert V_{B\_left} \rvert}{2 \cdot \mathrm{Area}}$ $f_v = 129.7$ psi $f_v < F'_v$	NDS 3.4 The shear is checked just to the left of the roller support at the point of maximum shear force. Shear checks in this example, but if it did not, the provisions of NDS 3.4.3.1 could be invoked. This section allows the shear to be calculated at a distance equal to the depth of the beam from the support.
<u>Use 5-inch by 9-5/8-in 20F-V5 glulam</u>	

Conclusion

Structural Wood Design Using ASD and LRFD is being developed as a companion design tool to the *2005 NDS*. This document will assist students and designers in understanding and applying new provisions of the *2005 NDS*, especially with respect to LRFD. For the designer steeped in the traditions of ASD, this will serve as an excellent resource to understand the straightforward approach to LRFD. For the new graduate, perhaps having been taught LRFD, but who may need to use ASD, the book will be an invaluable resource. Perhaps most importantly, it will also be an excellent tool to introduce new designers to the concepts of structural design with wood using either ASD or LRFD.

The author is Director of Technical Media with AF&PA's American Wood Council.