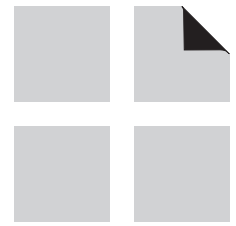


# BEAM DESIGN FORMULAS WITH SHEAR AND MOMENT DIAGRAMS



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ANSI  
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# NDS<sup>®</sup>

NATIONAL DESIGN SPECIFICATION<sup>®</sup>  
FOR WOOD CONSTRUCTION

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# BEAM FORMULAS WITH SHEAR AND MOMENT DIAGRAMS

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American Wood Council  
1111 19<sup>th</sup> St., NW, Suite 800  
Washington, DC 20036  
202-463-4713

[awcinfo@afandpa.org](mailto:awcinfo@afandpa.org)

[www.awc.org](http://www.awc.org)

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## Introduction

Figures 1 through 32 provide a series of shear and moment diagrams with accompanying formulas for design of beams under various static loading conditions.

Shear and moment diagrams and formulas are excerpted from the *Western Woods Use Book*, 4th edition, and are provided herein as a courtesy of [Western Wood Products Association](#).

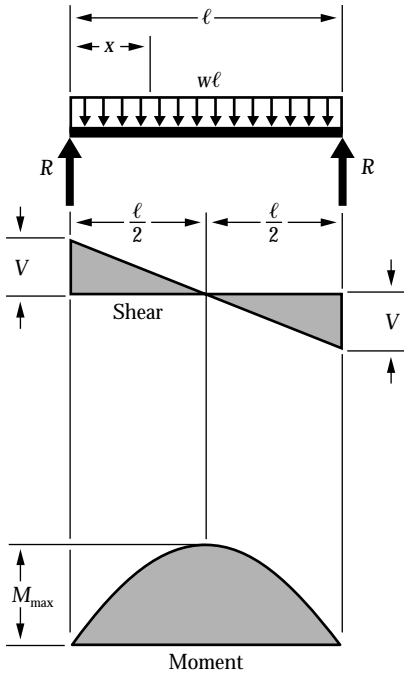
## Notations Relative to “Shear and Moment Diagrams”

$E$	= modulus of elasticity, psi
$I$	= moment of inertia, in. <sup>4</sup>
$L$	= span length of the bending member, ft.
$\ell$	= span length of the bending member, in.
$M$	= maximum bending moment, in.-lbs.
$P$	= total concentrated load, lbs.
$R$	= reaction load at bearing point, lbs.
$V$	= shear force, lbs.
$W$	= total uniform load, lbs.
$w$	= load per unit length, lbs./in.
$\Delta$	= deflection or deformation, in.
$x$	= horizontal distance from reaction to point on beam, in.

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**Figure 1 Simple Beam – Uniformly Distributed Load**



$$R = V \dots \dots \dots = \frac{w\ell}{2}$$

$$V_x \dots \dots \dots = w\left(\frac{\ell}{2} - x\right)$$

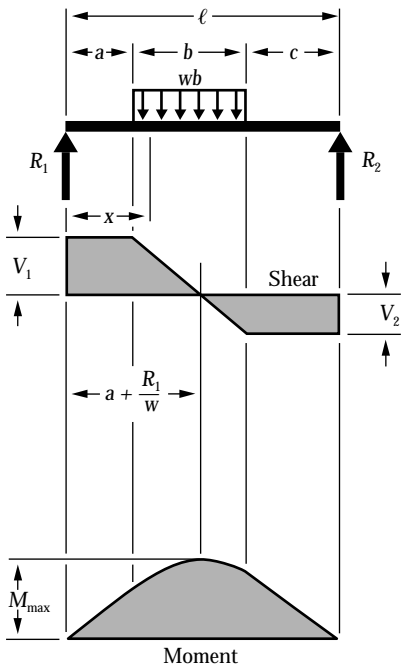
$$M_{\max} \text{ (at center)} \dots \dots \dots = \frac{w\ell^2}{8}$$

$$M_x \dots \dots \dots = \frac{wx}{2}(\ell - x)$$

$$\Delta_{\max} \text{ (at center)} \dots \dots \dots = \frac{5w\ell^4}{384EI}$$

$$\Delta_x \dots \dots \dots = \frac{wx}{24EI}(\ell^3 - 2\ell x^2 + x^3)$$

**Figure 2 Simple Beam – Uniform Load Partially Distributed**



$$R_1 = V_1 \text{ (max when } a < c) \dots \dots = \frac{wb}{2\ell}(2c + b)$$

$$R_2 = V_2 \text{ (max when } a > c) \dots \dots = \frac{wb}{2\ell}(2a + b)$$

$$V_x \text{ (when } x > a \text{ and } < (a + b)) \dots \dots = R_1 - w(x - a)$$

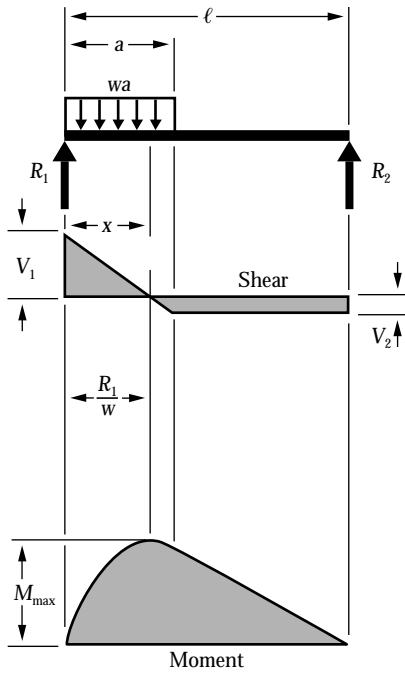
$$M_{\max} \left( \text{at } x = a + \frac{R_1}{w} \right) \dots \dots = R_1 \left( a + \frac{R_1}{2w} \right)$$

$$M_x \text{ (when } x < a) \dots \dots \dots = R_1 x$$

$$M_x \text{ (when } x > a \text{ and } < (a + b)) \dots \dots = R_1 x - \frac{w}{2}(x - a)^2$$

$$M_x \text{ (when } x > (a + b)) \dots \dots \dots = R_2(\ell - x)$$

**Figure 3 Simple Beam – Uniform Load Partially Distributed at One End**



$$R_1 = V_1 \dots \dots \dots = \frac{wa}{2\ell}(2\ell - a)$$

$$R_2 = V_2 \dots \dots \dots = \frac{wa^2}{2\ell}$$

$$V_x \text{ (when } x < a) \dots \dots \dots = R_1 - wx$$

$$M_{\max} \left( \text{at } x = \frac{R_1}{w} \right) \dots \dots \dots = \frac{R_1^2}{2w}$$

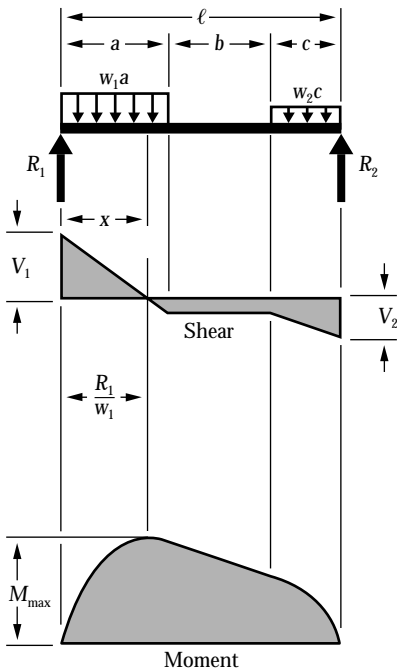
$$M_x \text{ (when } x < a) \dots \dots \dots = R_1x - \frac{wx^2}{2}$$

$$M_x \text{ (when } x > a) \dots \dots \dots = R_2(\ell - x)$$

$$\Delta_x \text{ (when } x < a) \dots \dots \dots = \frac{wx}{24 E I \ell} (a^2(2\ell - a)^2 - 2ax^2(2\ell - a) + \ell x^3)$$

$$\Delta_x \text{ (when } x > a) \dots \dots \dots = \frac{wa^2(\ell - x)}{24 E I \ell} (4x\ell - 2x^2 - a^2)$$

**Figure 4 Simple Beam – Uniform Load Partially Distributed at Each End**



$$R_1 = V_1 \dots \dots \dots = \frac{w_1a(2\ell - a) + w_2c^2}{2\ell}$$

$$R_2 = V_2 \dots \dots \dots = \frac{w_2c(2\ell - c) + w_1a^2}{2\ell}$$

$$V_x \text{ (when } x < a) \dots \dots \dots = R_1 - w_1x$$

$$V_x \text{ (when } x > a \text{ and } < (a + b)) \dots \dots \dots = R_1 - w_1a$$

$$V_x \text{ (when } x > (a + b)) \dots \dots \dots = R_2 - w_2(\ell - x)$$

$$M_{\max} \left( \text{at } x = \frac{R_1}{w_1} \text{ when } R_1 < w_1a \right) \dots \dots \dots = \frac{R_1^2}{2w_1}$$

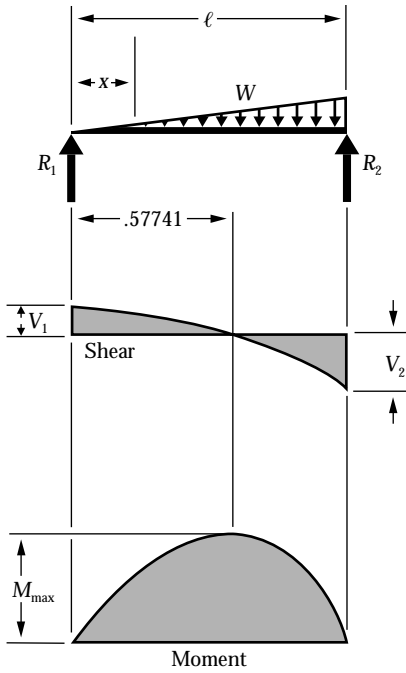
$$M_{\max} \left( \text{at } x = \ell - \frac{R_2}{w_2} \text{ when } R_2 < w_2c \right) = \frac{R_2^2}{2w_2}$$

$$M_x \text{ (when } x < a) \dots \dots \dots = R_1x - \frac{w_1x^2}{2}$$

$$M_x \text{ (when } x > a \text{ and } < (a + b)) \dots \dots \dots = R_1x - \frac{w_1a}{2}(2x - a)$$

$$M_x \text{ (when } x > (a + b)) \dots \dots \dots = R_2(\ell - x) - \frac{w_2(\ell - x)^2}{2}$$

**Figure 5 Simple Beam – Load Increasing Uniformly to One End**



$$R_1 = V_1 \dots \dots \dots = \frac{W}{3}$$

$$R_2 = V_2 \dots \dots \dots = \frac{2W}{3}$$

$$V_x \dots \dots \dots = \frac{W}{3} - \frac{Wx^2}{\ell^2}$$

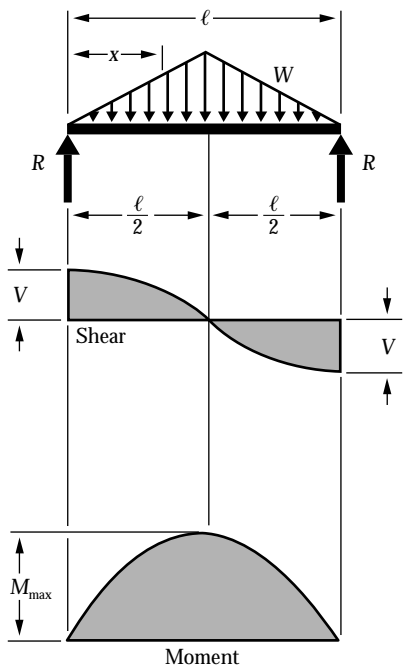
$$M_{max} \left( \text{at } x = \frac{\ell}{\sqrt{3}} = .5774\ell \right) \dots \dots = \frac{2W\ell}{9\sqrt{3}} = .1283W\ell$$

$$M_x \dots \dots \dots = \frac{Wx}{3\ell^2} (\ell^2 - x^2)$$

$$\Delta_{max} \left( \text{at } x = \ell \sqrt{1 - \sqrt{\frac{8}{15}}} = .5193\ell \right) \dots = .01304 \frac{W\ell^3}{EI}$$

$$\Delta_x \dots \dots \dots = \frac{Wx}{180EI\ell^2} (3x^4 - 10\ell^2x^2 + 7\ell^4)$$

**Figure 6 Simple Beam – Load Increasing Uniformly to Center**



$$R = V \dots \dots \dots = \frac{W}{2}$$

$$V_x \left( \text{when } x < \frac{\ell}{2} \right) \dots \dots \dots = \frac{W}{2\ell^2} (\ell^2 - 4x^2)$$

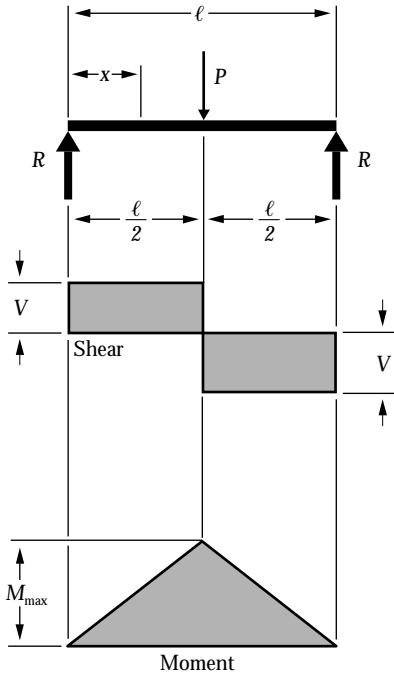
$$M_{max} \text{ (at center)} \dots \dots \dots = \frac{W\ell}{6}$$

$$M_x \left( \text{when } x < \frac{\ell}{2} \right) \dots \dots \dots = Wx \left( \frac{1}{2} - \frac{2x^2}{3\ell^2} \right)$$

$$\Delta_{max} \text{ (at center)} \dots \dots \dots = \frac{W\ell^3}{60EI}$$

$$\Delta_x \dots \dots \dots = \frac{Wx}{480EI\ell^2} (5\ell^2 - 4x^2)^2$$

**Figure 7 Simple Beam – Concentrated Load at Center**



$$R = V \dots\dots\dots = \frac{P}{2}$$

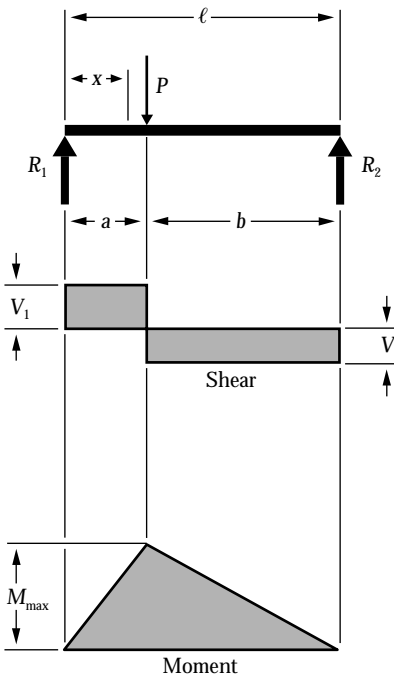
$$M_{\max} \text{ (at point of load)} \dots\dots\dots = \frac{P\ell}{4}$$

$$M_x \left( \text{when } x < \frac{\ell}{2} \right) \dots\dots\dots = \frac{Px}{2}$$

$$\Delta_{\max} \text{ (at point of load)} \dots\dots\dots = \frac{P\ell^3}{48EI}$$

$$\Delta_x \left( \text{when } x < \frac{\ell}{2} \right) \dots\dots\dots = \frac{Px}{48EI} (3\ell^2 - 4x^2)$$

**Figure 8 Simple Beam – Concentrated Load at Any Point**



$$R_1 = V_1 \text{ (max when } a < b) \dots\dots\dots = \frac{Pb}{\ell}$$

$$R_2 = V_2 \text{ (max when } a > b) \dots\dots\dots = \frac{Pa}{\ell}$$

$$M_{\max} \text{ (at point of load)} \dots\dots\dots = \frac{Pab}{\ell}$$

$$M_x \text{ (when } x < a) \dots\dots\dots = \frac{Pbx}{\ell}$$

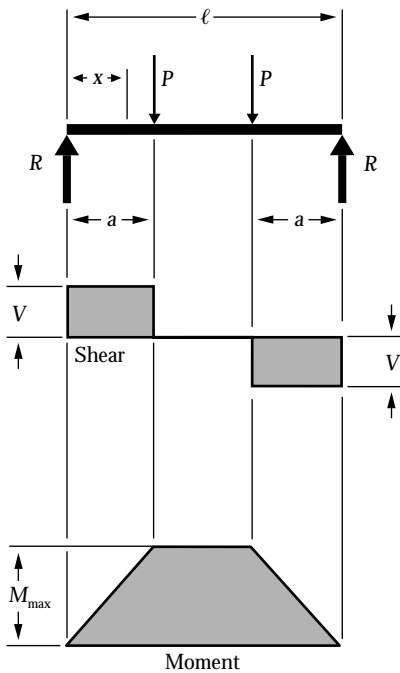
$$\Delta_{\max} \left( \text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b \right) \dots\dots\dots = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI\ell}$$

$$\Delta_a \text{ (at point of load)} \dots\dots\dots = \frac{Pa^2b^2}{3EI\ell}$$

$$\Delta_x \text{ (when } x < a) \dots\dots\dots = \frac{Pbx}{6EI\ell} (\ell^2 - b^2 - x^2)$$

$$\Delta_x \text{ (when } x > a) \dots\dots\dots = \frac{Pa(\ell-x)}{6EI\ell} (2\ell x - x^2 - a^2)$$

**Figure 9 Simple Beam – Two Equal Concentrated Loads Symmetrically Placed**



$$R = V \dots \dots \dots = P$$

$$M_{\max} \text{ (between loads)} \dots \dots \dots = Pa$$

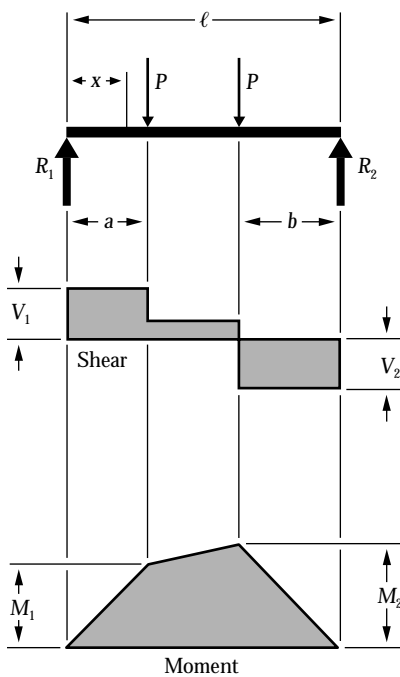
$$M_x \text{ (when } x < a) \dots \dots \dots = Px$$

$$\Delta_{\max} \text{ (at center)} \dots \dots \dots = \frac{Pa}{24EI} (3\ell^2 - 4a^2)$$

$$\Delta_x \text{ (when } x < a) \dots \dots \dots = \frac{Px}{6EI} (3\ell a - 3a^2 - x^2)$$

$$\Delta_x \text{ (when } x > a \text{ and } < (\ell - a)) \dots \dots = \frac{Pa}{6EI} (3\ell x - 3x^2 - a^2)$$

**Figure 10 Simple Beam – Two Equal Concentrated Loads Unsymmetrically Placed**



$$R_1 = V_1 \text{ (max when } a < b) \dots \dots \dots = \frac{P}{\ell} (\ell - a + b)$$

$$R_2 = V_2 \text{ (max when } a > b) \dots \dots \dots = \frac{P}{\ell} (\ell - b + a)$$

$$V_x \text{ (when } x > a \text{ and } < (\ell - b)) \dots \dots = \frac{P}{\ell} (b - a)$$

$$M_1 \text{ (max when } a > b) \dots \dots \dots = R_1 a$$

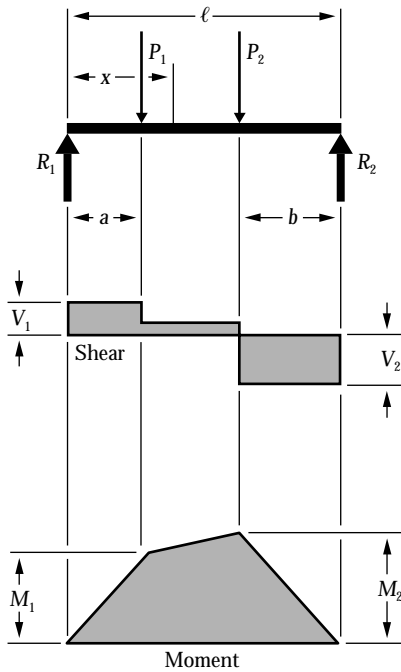
$$M_2 \text{ (max when } a < b) \dots \dots \dots = R_2 b$$

$$M_x \text{ (when } x < a) \dots \dots \dots = R_1 x$$

$$M_x \text{ (when } x > a \text{ and } < (\ell - b)) \dots \dots = R_1 x - P(x - a)$$



**Figure 11 Simple Beam – Two Unequal Concentrated Loads Unsymmetrically Placed**



$$R_1 = V_1 \dots \dots \dots = \frac{P_1(\ell - a) + P_2 b}{\ell}$$

$$R_2 = V_2 \dots \dots \dots = \frac{P_1 a + P_2(\ell - b)}{\ell}$$

$$V_x \text{ (when } x > a \text{ and } < (\ell - b)) \dots \dots = R_1 - P_1$$

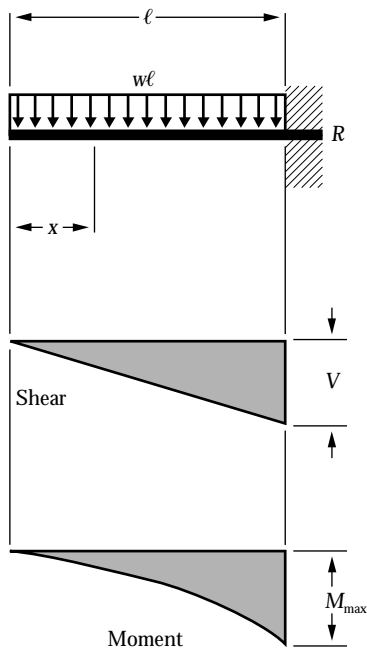
$$M_1 \text{ (max when } R_1 < P_1) \dots \dots \dots = R_1 a$$

$$M_2 \text{ (max when } R_2 < P_2) \dots \dots \dots = R_2 b$$

$$M_x \text{ (when } x < a) \dots \dots \dots = R_1 x$$

$$M_x \text{ (when } x > a \text{ and } < (\ell - b)) \dots \dots = R_1 x - P_1(x - a)$$

**Figure 12 Cantilever Beam – Uniformly Distributed Load**



$$R = V \dots \dots \dots = w\ell$$

$$V_x \dots \dots \dots = wx$$

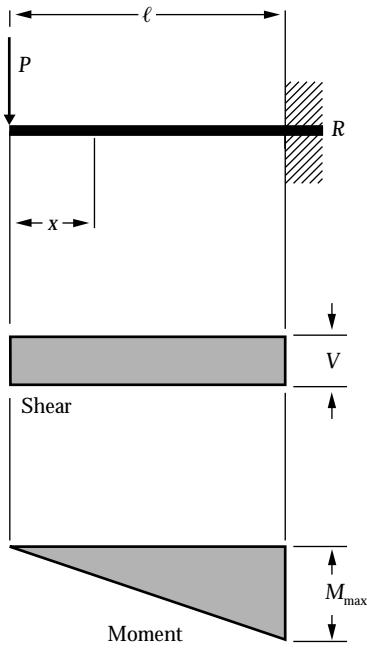
$$M_{\max} \text{ (at fixed end)} \dots \dots \dots = \frac{w\ell^2}{2}$$

$$M_x \dots \dots \dots = \frac{wx^2}{2}$$

$$\Delta_{\max} \text{ (at free end)} \dots \dots \dots = \frac{w\ell^4}{8EI}$$

$$\Delta_x \dots \dots \dots = \frac{w}{24EI} (x^4 - 4\ell^3 x + 3\ell^4)$$

**Figure 13 Cantilever Beam – Concentrated Load at Free End**



$$R = V \dots\dots\dots = P$$

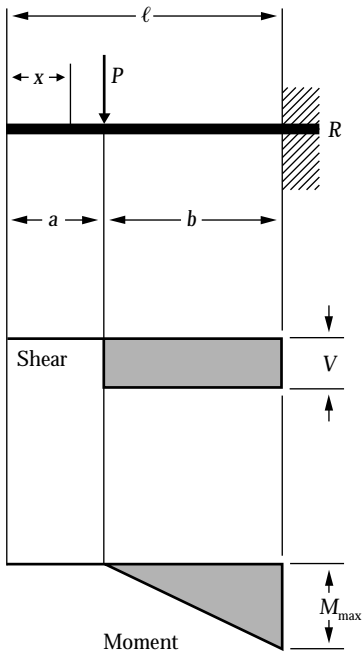
$$M_{\max} \text{ (at fixed end)} \dots\dots\dots = P\ell$$

$$M_x \dots\dots\dots = Px$$

$$\Delta_{\max} \text{ (at free end)} \dots\dots\dots = \frac{P\ell^3}{3EI}$$

$$\Delta_x \dots\dots\dots = \frac{P}{6EI} (2\ell^3 - 3\ell^2x + x^3)$$

**Figure 14 Cantilever Beam – Concentrated Load at Any Point**



$$R = V \dots\dots\dots = P$$

$$M_{\max} \text{ (at fixed end)} \dots\dots\dots = Pb$$

$$M_x \text{ (when } x > a) \dots\dots\dots = P(x - a)$$

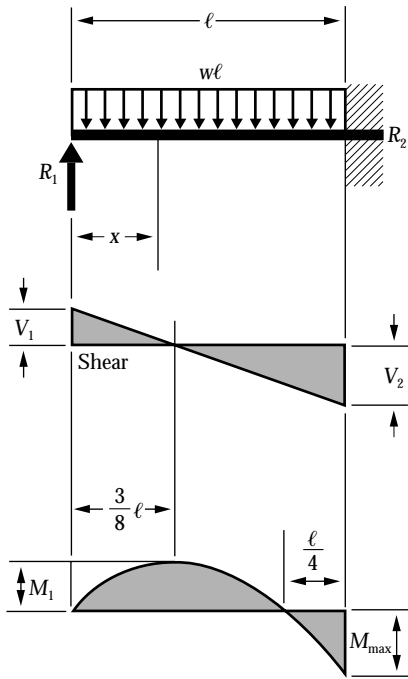
$$\Delta_{\max} \text{ (at free end)} \dots\dots\dots = \frac{Pb^2}{6EI} (3\ell - b)$$

$$\Delta_a \text{ (at point of load)} \dots\dots\dots = \frac{Pb^3}{3EI}$$

$$\Delta_x \text{ (when } x < a) \dots\dots\dots = \frac{Pb^2}{6EI} (3\ell - 3x - b)$$

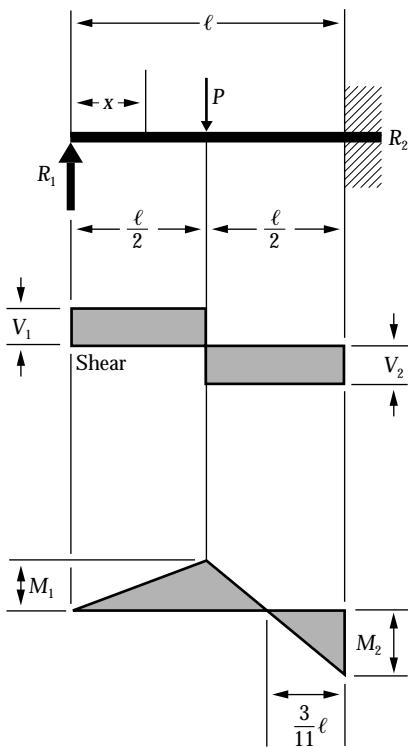
$$\Delta_x \text{ (when } x > a) \dots\dots\dots = \frac{P(\ell - x)^2}{6EI} (3b - \ell + x)$$

**Figure 15 Beam Fixed at One End, Supported at Other – Uniformly Distributed Load**



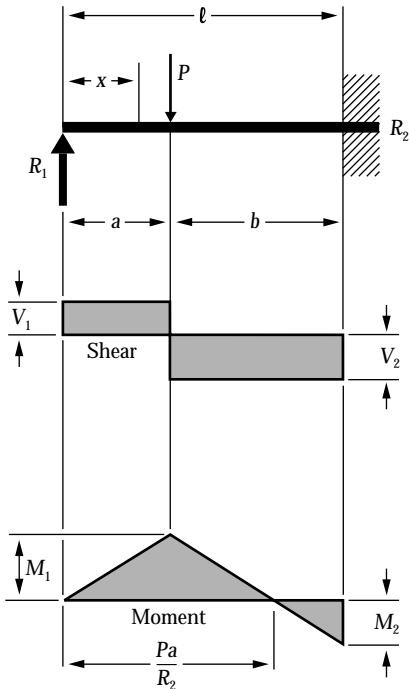
$$\begin{aligned}
 R_1 = V_1 & \dots \dots \dots = \frac{3w\ell}{8} \\
 R_2 = V_2 & \dots \dots \dots = \frac{5w\ell}{8} \\
 V_x & \dots \dots \dots = R_1 - wx \\
 M_{\max} & \dots \dots \dots = \frac{w\ell^2}{8} \\
 M_1 \left( \text{at } x = \frac{3}{8} \ell \right) & \dots \dots \dots = \frac{9}{128} w\ell^2 \\
 M_x & \dots \dots \dots = R_1 x - \frac{wx^2}{2} \\
 \Delta_{\max} \left( \text{at } x = \frac{\ell}{16} (1 + \sqrt{33}) = .4215 \ell \right) & \dots \dots \dots = \frac{w\ell^4}{185EI} \\
 \Delta_x & \dots \dots \dots = \frac{wx}{48EI} (\ell^3 - 3\ell x^2 + 2x^3)
 \end{aligned}$$

**Figure 16 Beam Fixed at One End, Supported at Other – Concentrated Load at Center**



$$\begin{aligned}
 R_1 = V_1 & \dots \dots \dots = \frac{5P}{16} \\
 R_2 = V_2 & \dots \dots \dots = \frac{11P}{16} \\
 M_{\max} \text{ (at fixed end)} & \dots \dots \dots = \frac{3P\ell}{16} \\
 M_1 \text{ (at point of load)} & \dots \dots \dots = \frac{5P\ell}{32} \\
 M_x \left( \text{when } x < \frac{\ell}{2} \right) & \dots \dots \dots = \frac{5Px}{16} \\
 M_x \left( \text{when } x > \frac{\ell}{2} \right) & \dots \dots \dots = P \left( \frac{\ell}{2} - \frac{11x}{16} \right) \\
 \Delta_{\max} \left( \text{at } x = \ell \sqrt{\frac{1}{5}} = .4472\ell \right) & \dots \dots \dots = \frac{P\ell^3}{48EI\sqrt{5}} = .009317 \frac{P\ell^3}{EI} \\
 \Delta_x \text{ (at point of load)} & \dots \dots \dots = \frac{7P\ell^3}{768EI} \\
 \Delta_x \left( \text{when } x < \frac{\ell}{2} \right) & \dots \dots \dots = \frac{Px}{96EI} (3\ell^2 - 5x^2) \\
 \Delta_x \left( \text{when } x > \frac{\ell}{2} \right) & \dots \dots \dots = \frac{P}{96EI} (x - \ell)^2 (11x - 2\ell)
 \end{aligned}$$

**Figure 17 Beam Fixed at One End, Supported at Other – Concentrated Load at Any Point**



$$R_1 = V_1 \dots \dots \dots = \frac{Pb^2}{2\ell^3}(a + 2\ell)$$

$$R_2 = V_2 \dots \dots \dots = \frac{Pa}{2\ell^3}(3\ell^2 - a^2)$$

$$M_1 \text{ (at point of load)} \dots \dots \dots = R_1 a$$

$$M_2 \text{ (at fixed end)} \dots \dots \dots = \frac{Pab}{2\ell^2}(a + \ell)$$

$$M_x \text{ (when } x < a) \dots \dots \dots = R_1 x$$

$$M_x \text{ (when } x > a) \dots \dots \dots = R_1 x - P(x - a)$$

$$\Delta_{\max} \left( \text{when } a < .414\ell \text{ at } x = \ell \frac{\ell^2 + a^2}{3\ell^2 - a^2} \right) = \frac{Pa}{3EI} \frac{(\ell^2 - a^2)^3}{(3\ell^2 - a^2)^2}$$

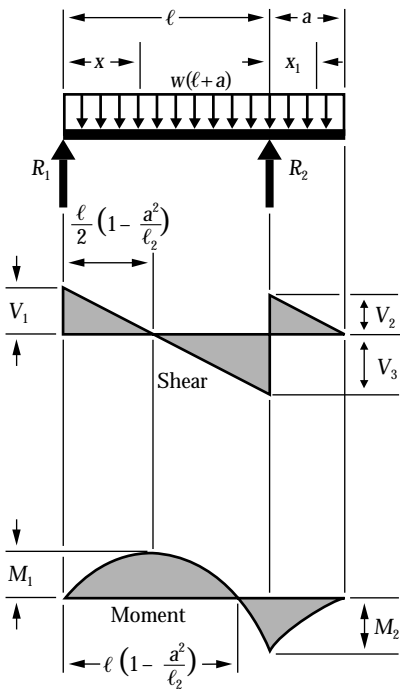
$$\Delta_{\max} \left( \text{when } a > .414\ell \text{ at } x = \ell \sqrt{\frac{a}{2\ell + a}} \right) = \frac{Pab^2}{6EI} \sqrt{\frac{a}{2\ell + a}}$$

$$\Delta_a \text{ (at point of load)} \dots \dots \dots = \frac{Pa^2 b^3}{12EI\ell^3}(3\ell + a)$$

$$\Delta_x \text{ (when } x < a) \dots \dots \dots = \frac{Pb^2 x}{12EI\ell^3}(3a\ell^2 - 2\ell x^2 - ax^2)$$

$$\Delta_x \text{ (when } x > a) \dots \dots \dots = \frac{Pa}{12EI\ell^3}(\ell - x)^2(3\ell^2 x - a^2 x - 2a^2 \ell)$$

**Figure 18 Beam Overhanging One Support – Uniformly Distributed Load**



$$R_1 = V_1 \dots \dots \dots = \frac{w}{2\ell}(\ell^2 - a^2)$$

$$R_2 = V_2 + V_3 \dots \dots \dots = \frac{w}{2\ell}(\ell + a)^2$$

$$V_2 \dots \dots \dots = wa$$

$$V_3 \dots \dots \dots = \frac{w}{2\ell}(\ell^2 + a^2)$$

$$V_x \text{ (between supports)} \dots \dots \dots = R_1 - wx$$

$$V_{x_1} \text{ (for overhang)} \dots \dots \dots = w(a - x_1)$$

$$M_1 \left( \text{at } x = \frac{\ell}{2} \left[ 1 - \frac{a^2}{\ell^2} \right] \right) \dots \dots \dots = \frac{w}{8\ell^2}(\ell + a)^2(\ell - a)^2$$

$$M_2 \text{ (at } R_2) \dots \dots \dots = \frac{wa^2}{2}$$

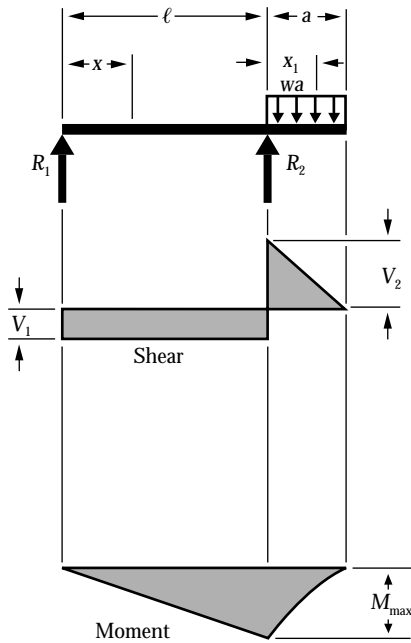
$$M_x \text{ (between supports)} \dots \dots \dots = \frac{wx}{2\ell}(\ell^2 - a^2 - x\ell)$$

$$M_{x_1} \text{ (for overhang)} \dots \dots \dots = \frac{w}{2}(a - x_1)^2$$

$$\Delta_x \text{ (between supports)} \dots \dots \dots = \frac{wx}{24EI\ell}(\ell^4 - 2\ell^2 x^2 + \ell x^3 - 2a^2 \ell^2 + 2a^2 x^2)$$

$$\Delta_{x_1} \text{ (for overhang)} \dots \dots \dots = \frac{wx_1}{24EI}(4a^2 \ell - \ell^3 + 6a^2 x_1 - 4ax_1^2 + x_1^3)$$

**Figure 19 Beam Overhanging One Support – Uniformly Distributed Load on Overhang**



$$R_1 = V_1 \dots \dots \dots = \frac{wa^2}{2\ell}$$

$$R_2 = V_1 + V_2 \dots \dots \dots = \frac{wa}{2\ell}(2\ell + a)$$

$$V_2 \dots \dots \dots = wa$$

$$V_{x_1} \text{ (for overhang)} \dots \dots \dots = w(a - x_1)$$

$$M_{max} \text{ (at } R_2) \dots \dots \dots = \frac{wa^2}{2}$$

$$M_x \text{ (between supports)} \dots \dots \dots = \frac{wa^2x}{2\ell}$$

$$M_{x_1} \text{ (for overhang)} \dots \dots \dots = \frac{w}{2}(a - x_1)^2$$

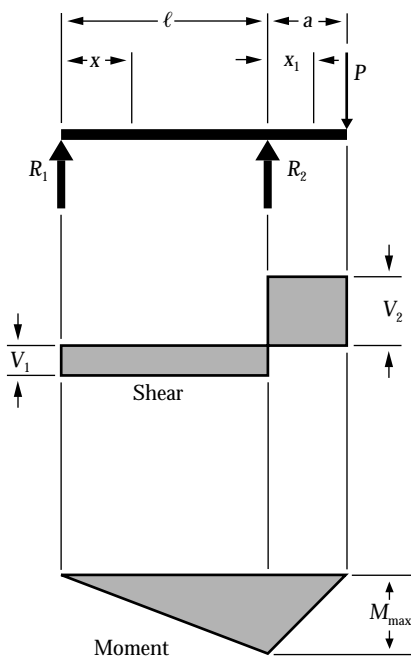
$$\Delta_{max} \left( \text{between supports at } x = \frac{\ell}{\sqrt{3}} \right) = \frac{wa^2\ell^2}{18\sqrt{3}EI} = .03208 \frac{wa^2\ell^2}{EI}$$

$$\Delta_{max} \text{ (for overhang at } x_1 = a) \dots \dots = \frac{wa^3}{24EI}(4\ell + 3a)$$

$$\Delta_x \text{ (between supports)} \dots \dots \dots = \frac{wa^2x}{12EI}(\ell^2 - x^2)$$

$$\Delta_{x_1} \text{ (for overhang)} \dots \dots \dots = \frac{wx_1}{24EI}(4a^2\ell + 6a^2x_1 - 4ax_1^2 + x_1^3)$$

**Figure 20 Beam Overhanging One Support – Concentrated Load at End of Overhang**



$$R_1 = V_1 \dots \dots \dots = \frac{Pa}{\ell}$$

$$R_2 = V_1 + V_2 \dots \dots \dots = \frac{P}{\ell}(\ell + a)$$

$$V_2 \dots \dots \dots = P$$

$$M_{max} \text{ (at } R_2) \dots \dots \dots = Pa$$

$$M_x \text{ (between supports)} \dots \dots \dots = \frac{Pax}{\ell}$$

$$M_{x_1} \text{ (for overhang)} \dots \dots \dots = P(a - x_1)$$

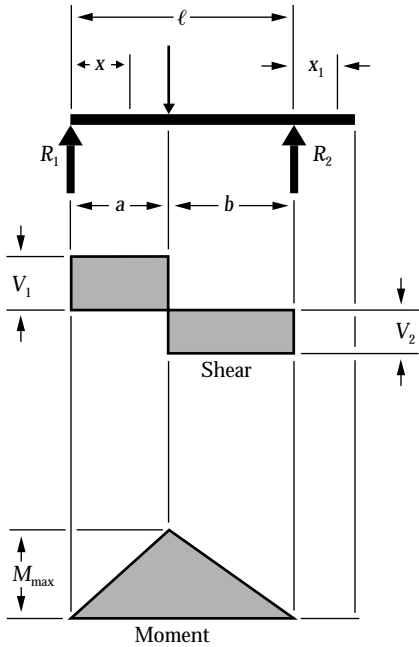
$$\Delta_{max} \left( \text{between supports at } x = \frac{\ell}{\sqrt{3}} \right) = \frac{Pal^2}{9\sqrt{3}EI} = .06415 \frac{Pal^2}{EI}$$

$$\Delta_{max} \text{ (for overhang at } x_1 = a) \dots \dots = \frac{Pa^2}{3EI}(\ell + a)$$

$$\Delta_x \text{ (between supports)} \dots \dots \dots = \frac{Pax}{6EI}(\ell^2 - x^2)$$

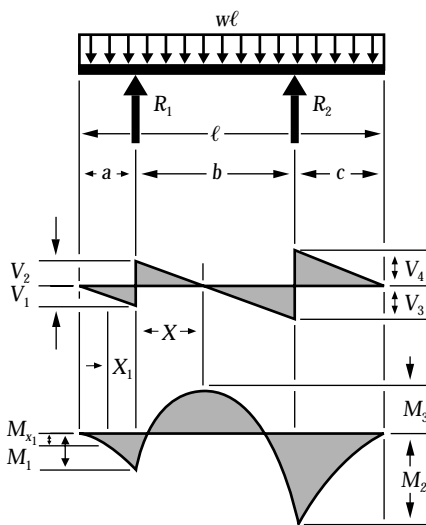
$$\Delta_{x_1} \text{ (for overhang)} \dots \dots \dots = \frac{Px_1}{6EI}(2a\ell + 3ax_1 - x_1^2)$$

**Figure 21 Beam Overhanging One Support – Concentrated Load at Any Point Between Supports**



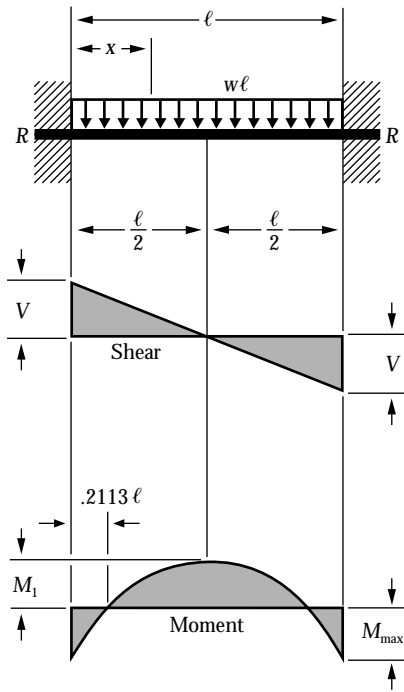
$$\begin{aligned}
 R_1 = V_1 \text{ (max when } a < b) & \dots \dots \dots = \frac{Pb}{\ell} \\
 R_2 = V_2 \text{ (max when } a > b) & \dots \dots \dots = \frac{Pa}{\ell} \\
 M_{\max} \text{ (at point of load)} & \dots \dots \dots = \frac{Pab}{\ell} \\
 M_x \text{ (when } x < a) & \dots \dots \dots = \frac{Pbx}{\ell} \\
 \Delta_{\max} \left( \text{at } x = \sqrt{\frac{a(a+2b)}{3}} \text{ when } a > b \right) & \dots \dots \dots = \frac{Pab(a+2b)\sqrt{3a(a+2b)}}{27EI\ell} \\
 \Delta_a \text{ (at point of load)} & \dots \dots \dots = \frac{Pa^2b^2}{3EI\ell} \\
 \Delta_x \text{ (when } x < a) & \dots \dots \dots = \frac{Pbx}{6EI\ell} (\ell^2 - b^2 - x^2) \\
 \Delta_x \text{ (when } x > a) & \dots \dots \dots = \frac{Pa(\ell - x)}{6EI\ell} (2\ell x - x^2 - a^2) \\
 \Delta_{x_1} & \dots \dots \dots = \frac{Pabx_1}{6EI\ell} (\ell + a)
 \end{aligned}$$

**Figure 22 Beam Overhanging Both Supports – Unequal Overhangs – Uniformly Distributed Load**



$$\begin{aligned}
 R_1 & \dots \dots \dots = \frac{w\ell(\ell - 2c)}{2b} \\
 R_2 & \dots \dots \dots = \frac{w\ell(\ell - 2a)}{2b} \\
 V_1 & \dots \dots \dots = wa \\
 V_2 & \dots \dots \dots = R_1 - V_1 \\
 V_3 & \dots \dots \dots = R_2 - V_4 \\
 V_4 & \dots \dots \dots = wc \\
 V_{x_1} & \dots \dots \dots = V_1 - wx_1 \\
 V_x \text{ (when } x < \ell) & \dots \dots \dots = R_1 - w(a + x_1) \\
 V_m \text{ (when } a < c) & \dots \dots \dots = R_2 - wc \\
 M_1 & \dots \dots \dots = -\frac{wa^2}{2} \\
 M_2 & \dots \dots \dots = -\frac{wc^2}{2} \\
 M_3 & \dots \dots \dots = R_1 \left( \frac{R_1}{2w} - a \right) \\
 M_x \left( \text{max when } x = \frac{R_1}{w} - a \right) & \dots \dots \dots = R_1 x - \frac{w(a+x)^2}{2}
 \end{aligned}$$

**Figure 23 Beam Fixed at Both Ends – Uniformly Distributed Load**



$$R = V \dots \dots \dots = \frac{wl}{2}$$

$$V_x \dots \dots \dots = w\left(\frac{\ell}{2} - x\right)$$

$$M_{\max} \text{ (at ends)} \dots \dots \dots = \frac{wl^2}{12}$$

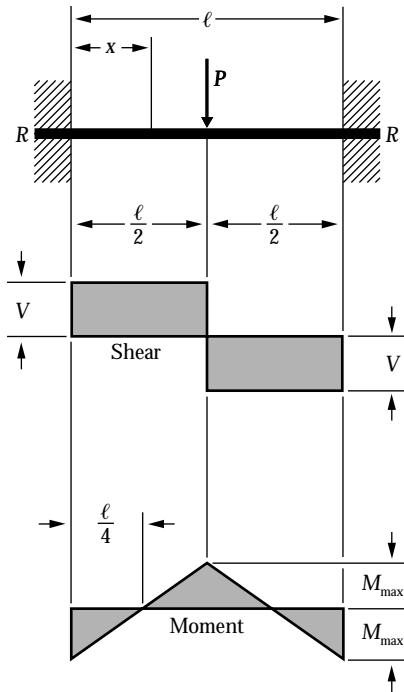
$$M_1 \text{ (at center)} \dots \dots \dots = \frac{wl^2}{24}$$

$$M_x \dots \dots \dots = \frac{w}{12}(6\ell x - \ell^2 - 6x^2)$$

$$\Delta_{\max} \text{ (at center)} \dots \dots \dots = \frac{wl^4}{384EI}$$

$$\Delta_x \dots \dots \dots = \frac{wx^2}{24EI}(\ell - x)^2$$

**Figure 24 Beam Fixed at Both Ends – Concentrated Load at Center**



$$R = V \dots \dots \dots = \frac{P}{2}$$

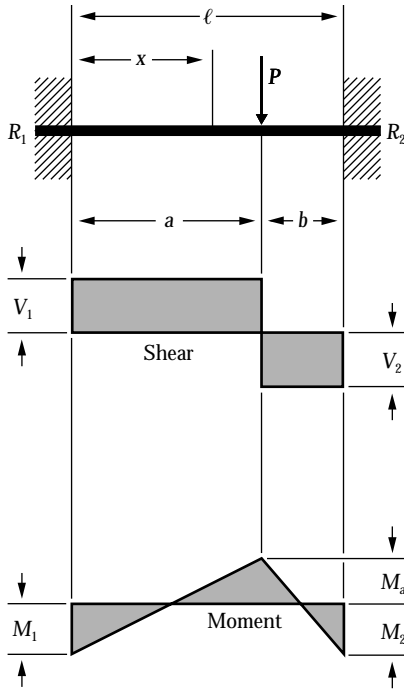
$$M_{\max} \text{ (at center and ends)} \dots \dots \dots = \frac{P\ell}{8}$$

$$M_x \left( \text{when } x < \frac{\ell}{2} \right) \dots \dots \dots = \frac{P}{8}(4x - \ell)$$

$$\Delta_{\max} \text{ (at center)} \dots \dots \dots = \frac{P\ell^3}{192EI}$$

$$\Delta_x \left( \text{when } x < \frac{\ell}{2} \right) \dots \dots \dots = \frac{Px^2}{48EI}(3\ell - 4x)$$

**Figure 25 Beam Fixed at Both Ends – Concentrated Load at Any Point**



$$R_1 = V_1 \text{ (max when } a < b) \dots \dots \dots = \frac{Pb^2}{\ell^3} (3a + b)$$

$$R_2 = V_2 \text{ (max when } a > b) \dots \dots \dots = \frac{Pa^2}{\ell^3} (a + 3b)$$

$$M_1 \text{ (max when } a < b) \dots \dots \dots = \frac{Pab^2}{\ell^2}$$

$$M_2 \text{ (max when } a > b) \dots \dots \dots = \frac{Pa^2b}{\ell^2}$$

$$M_a \text{ (at point of load) } \dots \dots \dots = \frac{2Pa^2b^2}{\ell^3}$$

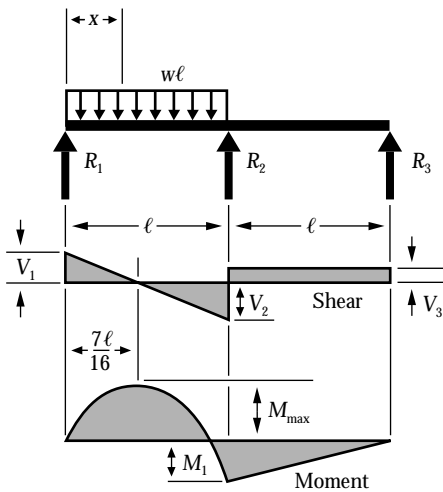
$$M_x \text{ (when } x < a) \dots \dots \dots = R_1x - \frac{Pbx}{\ell^2}$$

$$\Delta_{\max} \left( \text{when } a > b \text{ at } x = \frac{2a\ell}{3a + b} \right) \dots \dots = \frac{2Pa^3b^2}{3EI(3a + b)^2}$$

$$\Delta_a \text{ (at point of load) } \dots \dots \dots = \frac{Pa^3b^3}{3EI\ell^3}$$

$$\Delta_x \text{ (when } x < a) \dots \dots \dots = \frac{Pb^2x^2}{6EI\ell^3} (3a\ell - 3ax - bx)$$

**Figure 26 Continuous Beam – Two Equal Spans – Uniform Load on One Span**



$$R_1 = V_1 \dots \dots \dots = \frac{7}{16} w\ell$$

$$R_2 = V_2 + V_3 \dots \dots \dots = \frac{5}{8} w\ell$$

$$R_3 = V_3 \dots \dots \dots = -\frac{1}{16} w\ell$$

$$V_2 \dots \dots \dots = \frac{9}{16} w\ell$$

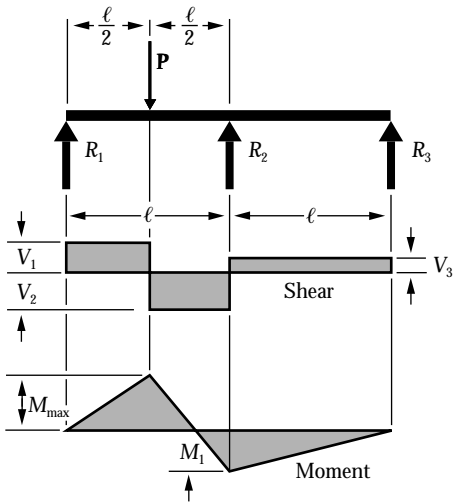
$$M_{\max} \left( \text{at } x = \frac{7}{16} \ell \right) \dots \dots \dots = \frac{49}{512} w\ell^2$$

$$M_1 \text{ (at support } R_2) \dots \dots \dots = \frac{1}{16} w\ell^2$$

$$M_x \text{ (when } x < \ell) \dots \dots \dots = \frac{wx}{16} (7\ell - 8x)$$



**Figure 27 Continuous Beam – Two Equal Spans – Concentrated Load at Center of One Span**



$$R_1 = V_1 \dots\dots\dots = \frac{13}{32} P$$

$$R_2 = V_2 + V_3 \dots\dots\dots = \frac{11}{16} P$$

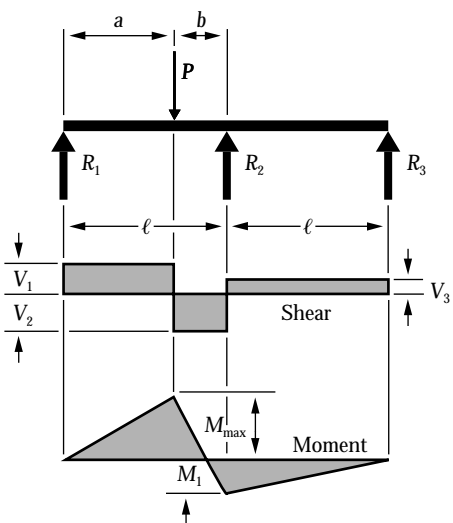
$$R_3 = V_3 \dots\dots\dots = -\frac{3}{32} P$$

$$V_2 \dots\dots\dots = \frac{19}{32} P$$

$$M_{\max} \text{ (at point of load)} \dots\dots\dots = \frac{13}{64} P\ell$$

$$M_1 \text{ (at support } R_2) \dots\dots\dots = \frac{3}{32} P\ell$$

**Figure 28 Continuous Beam – Two Equal Spans – Concentrated Load at Any Point**



$$R_1 = V_1 \dots\dots\dots = \frac{Pb}{4\ell^3} (4\ell^2 - a(\ell + a))$$

$$R_2 = V_2 + V_3 \dots\dots\dots = \frac{Pa}{2\ell^3} (2\ell^2 + b(\ell + a))$$

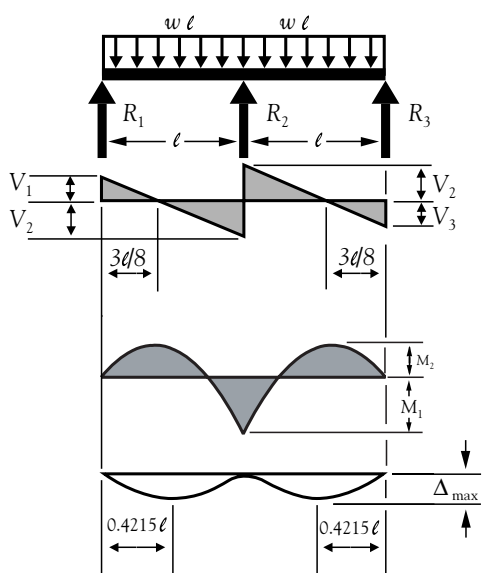
$$R_3 = V_3 \dots\dots\dots = -\frac{Pab}{4\ell^3} (\ell + a)$$

$$V_2 \dots\dots\dots = \frac{Pa}{4\ell^3} (4\ell^2 + b(\ell + a))$$

$$M_{\max} \text{ (at point of load)} \dots\dots\dots = \frac{Pab}{4\ell^3} (4\ell^2 - a(\ell + a))$$

$$M_1 \text{ (at support } R_2) \dots\dots\dots = \frac{Pab}{4\ell^2} (\ell + a)$$

**Figure 29 Continuous Beam – Two Equal Spans – Uniformly Distributed Load**



$$R_1 = V_1 = R_3 = V_3 \dots \dots \dots = \frac{3w\ell}{8}$$

$$R_2 \dots \dots \dots = \frac{10w\ell}{8}$$

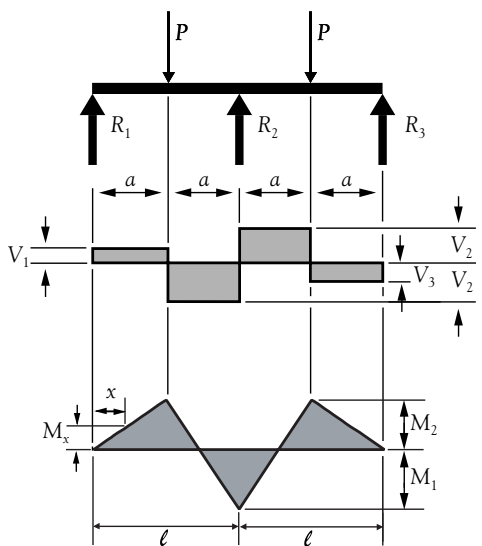
$$V_2 = V_{\max} \dots \dots \dots = \frac{5w\ell}{8}$$

$$M_1 \dots \dots \dots = \frac{w\ell^2}{8}$$

$$M_2 \left( \text{at } \frac{3\ell}{8} \right) \dots \dots \dots = \frac{9w\ell^2}{128}$$

$$\Delta_{\max} \left( \text{at } 0.4215\ell, \text{ approx. from } R_1 \text{ and } R_3 \right) \dots \dots = \frac{w\ell^4}{185EI}$$

**Figure 30 Continuous Beam – Two Equal Spans – Two Equal Concentrated Loads Symmetrically Placed**



$$R_1 = V_1 = R_3 = V_3 \dots \dots \dots = \frac{5P}{16}$$

$$R_2 = 2V_2 \dots \dots \dots = \frac{11P}{8}$$

$$V_2 = P - R_1 \dots \dots \dots = \frac{11P}{16}$$

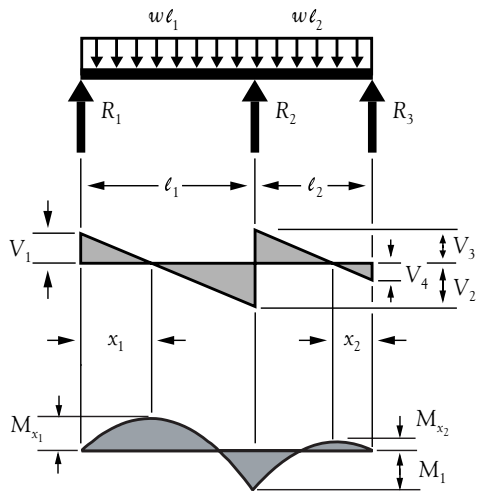
$$V_{\max} \dots \dots \dots = V_2$$

$$M_1 \dots \dots \dots = -\frac{3P\ell}{16}$$

$$M_2 \dots \dots \dots = \frac{5P\ell}{32}$$

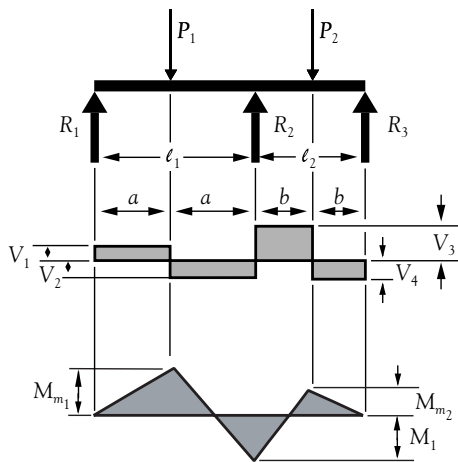
$$M_x \text{ (when } x < a) \dots \dots \dots = R_1x$$

**Figure 31 Continuous Beam – Two Unequal Spans – Uniformly Distributed Load**



$$\begin{aligned}
 R_1 & \dots \dots \dots = \frac{M_1}{\ell_1} + \frac{w\ell_1}{2} \\
 R_2 & \dots \dots \dots = w\ell_1 + w\ell_2 - R_1 - R_3 \\
 R_3 = V_4 & \dots \dots \dots = \frac{M_1}{\ell_2} + \frac{w\ell_2}{2} \\
 V_1 & \dots \dots \dots = R_1 \\
 V_2 & \dots \dots \dots = w\ell_1 - R_1 \\
 V_3 & \dots \dots \dots = w\ell_2 - R_3 \\
 V_4 & \dots \dots \dots = R_3 \\
 M_1 & \dots \dots \dots = -\frac{w\ell_2^3 + w\ell_1^3}{8(\ell_1 + \ell_2)} \\
 M_{x_1} \left( \text{when } x_1 = \frac{R_1}{w} \right) & \dots \dots \dots = R_1 x_1 - \frac{w x_1^2}{2} \\
 M_{x_2} \left( \text{when } x_2 = \frac{R_3}{w} \right) & \dots \dots \dots = R_3 x_2 - \frac{w x_2^2}{2}
 \end{aligned}$$

**Figure 32 Continuous Beam – Two Unequal Spans – Concentrated Load on Each Span Symmetrically Placed**



$$\begin{aligned}
 R_1 & \dots \dots \dots = \frac{M_1}{\ell_1} + \frac{P_1}{2} \\
 R_2 & \dots \dots \dots = P_1 + P_2 - R_1 - R_3 \\
 R_3 & \dots \dots \dots = \frac{M_1}{\ell_2} + \frac{P_2}{2} \\
 V_1 & \dots \dots \dots = R_1 \\
 V_2 & \dots \dots \dots = P_1 - R_1 \\
 V_3 & \dots \dots \dots = P_2 - R_3 \\
 V_4 & \dots \dots \dots = R_3 \\
 M_1 & \dots \dots \dots = -\frac{3}{16} \left( \frac{P_1 \ell_1^2 + P_2 \ell_2^2}{\ell_1 + \ell_2} \right) \\
 M_{m_1} & \dots \dots \dots = R_1 a \\
 M_{m_2} & \dots \dots \dots = R_3 b
 \end{aligned}$$

**American Forest & Paper Association  
American Wood Council  
1111 19th Street, NW  
Suite 800  
Washington, DC 20036  
Phone: 202-463-4713  
Fax: 202-463-2791  
awcinfo@afandpa.org  
www.awc.org**

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