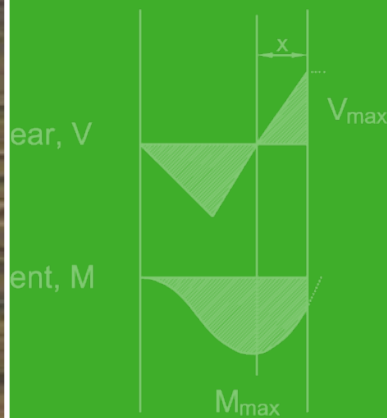
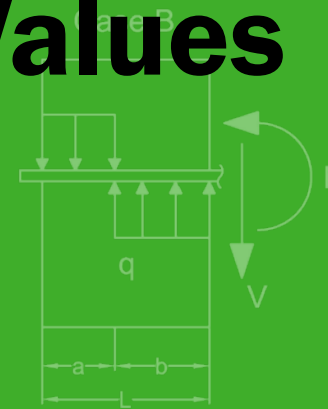
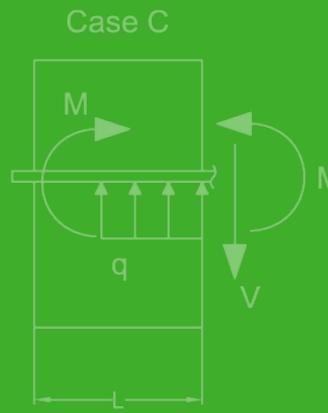
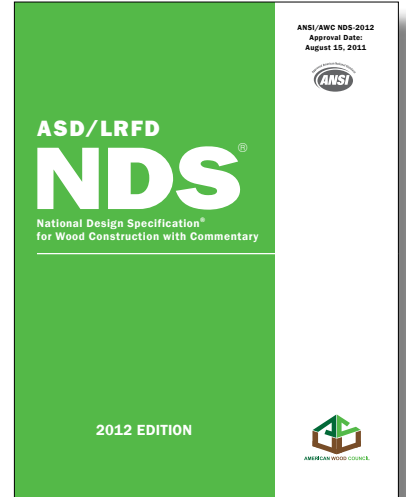


# TECHNICAL REPORT 12

# General Dowel Equations for Calculating Lateral Connection Values with Appendix A



$$qx = V$$
$$qa^2 = M_{max}$$



AMERICAN WOOD COUNCIL

**The American Wood Council (AWC)** is the voice of North American traditional and engineered wood products. From a renewable resource that absorbs and sequesters carbon, the wood products industry makes products that are essential to everyday life. AWC's engineers, technologists, scientists, and building code experts develop state-of-the-art engineering data, technology, and standards on structural wood products for use by design professionals, building officials, and wood products manufacturers to assure the safe and efficient design and use of wood structural components.

## **TECHNICAL REPORT 12**

---

# **General Dowel Equations for Calculating Lateral Connection Values**

## **with Appendix A**

While every effort has been made to insure the accuracy of the information presented, and special effort has been made to assure that the information reflects the state-of-the-art, neither the American Wood Council nor its members assume any responsibility for any particular design prepared from this publication. Those using this document assume all liability from its use.

Copyright © 2015  
American Wood Council  
[info@awc.org](mailto:info@awc.org)  
[www.awc.org](http://www.awc.org)



AMERICAN WOOD COUNCIL

# Table of Contents

Part/Title	Page	Part/Title	Page
<b>1 General Dowel Equations</b>		<b>3 Example Problems</b>	
1.1 Introduction	1	3.1 Bolted Connection with Gap	17
1.2 Lateral Connection Values	1	3.2 Lag Screw Connection	17
1.3 Reference Design Value	1	3.3 Lag Screw Penetration	20
1.4 Connection with Members of Solid Cross Section	1	3.4 Steel Side Member to Wood Main Member – Tapered Tip Effects	22
1.5 Connections with Members of Hollow Cross Section	1	3.5 Bolted Connection through Solid and Hollow Members – Double Shear	23
1.6 Dowel Bearing Strength and Fastener Bending Yield Strength	2	<b>4 References</b>	
1.7 Notation	2	References	24
<b>2 Equation Derivation</b>		<b>Appendix A Dowel Bearing Strength and Fastener Bending Yield Strength</b>	
2.1 Introduction	6	A.1 Introduction	25
2.2 Equation Derivation for Connections with Members of Solid Cross Section	7	A.2 Notation	25
2.3 Tapered Tip Equation Derivation	13	A.3 Dowel Bearing Strength	25
2.4 Equation Derivation for Connections with Members of Hollow Cross Section	14	A.4 Dowel Bending Yield Strength	28
		A.5 References	29

## List of Tables

1-1 General Dowel Equations for Solid Cross Section Members.....	3	A1 Dowel Bearing Strength, $F_c$ .....	27
1-2 General Dowel Equations for Solid Cross Section Main Members and Hollow Cross Section Side Member(s).....	4	A2 Dowel Bending Yield Strength.....	29
1-3 General Dowel Equations for Hollow Cross Section Main Member and Solid Cross Section Side Member(s).....	5		

## List of Figures

2-1 Connection Yield Modes – Solid Cross Section Members.....	6	2-5 Single Shear Connection – Mode II .....	12
2-2 Connection Yield Modes – Solid and Hollow Cross Section Members.....	7	2-6 Dowel Bearing with Tapered Tip Connector .....	13
2-3 General Conditions of Dowel Loading – Solid Members .....	8	2-7 General Conditions of Dowel Loading – Hollow Cross Section Members .	15
2-4 Connection Yield Modes Assumed Loading – Solid Members .....	11	A1 Dowel Bearing Strength Test Specimen.....	26
		A2 Dowel Bending Strength .....	28

## PART 1. General Dowel Equations

### 1.1 Introduction

A generalized form of the *National Design Specification*<sup>®</sup> (*NDS*<sup>®</sup>) for *Wood Construction* (AWC, 2012) yield limit equations applicable to dowel-type fastener connections between members of solid cross section is provided in Table 1-1 of this report. Equations are also provided for connections with side or main members of hollow cross section, in Tables 1-2 and 1-3, respectively. The variables for these tables are defined in Section 1.6. These general dowel equations are applicable to NDS connection conditions, allow for evaluation of connections with gaps between connected members, and include separate equation variables to better account for varying bearing and moment resistances of fasteners having more than one diameter along their length (such as threaded fasteners).

### 1.2 Lateral Connection Values

Yield limit equations provided herein apply to calculation of lateral values for single fastener connections between wood-based members and connections of wood-based members to steel and concrete/masonry components. Design criteria for dowel type fastener connections are provided in the *NDS for Wood Construction* and include the following considerations:

- fastener design limit states (e.g. tension, bearing, and shear);
- fastener spacing, edge, and end distance;
- connection fabrication and tolerances;
- connection geometry;
- multiple fasteners and group action;
- member strength at the connection;
- adjustments for end use (e.g. load duration and wet service) and fastener type (e.g. drift bolts and drift pins).
- member bearing strengths;
- fastener bending strengths;
- reduction factors,  $R_d$ , to adjust reference 5% offset yield connection values,  $P$ , to reference allowable stress design values,  $Z$ .

### 1.3 Reference Design Value

The reference design value,  $Z$ , is the minimum of the calculated,  $P/R_d$ , for all yield modes as follows:

$$Z = \text{Minimum value of } \left( \frac{P}{R_d} \right) \quad (1)$$

where:

$Z$  = Reference lateral design value

$P$  = Reference 5% offset yield value

$R_d$  = NDS reduction term for dowel type fastener connections, see NDS Table 11.3.1B

### 1.4 Connections with Members of Solid Cross Section

Equations in Table 1-1 are applicable for calculation of reference 5% offset yield,  $P$ , for dowel type fastener connections where connected members are of solid cross section. The reference 5% offset yield,  $P$ , is calculated for each applicable connection yield mode, as depicted in Table 1-1. Example applications of equations in Table 1-1 are provided in Part 3 of this report.

### 1.5 Connections with Members of Hollow Cross Section

Equations in Table 1-2 are applicable for calculation of reference 5% offset yield,  $P$ , for dowel type fastener connections where the main member is of solid cross section and the side member(s) have a hollow cross section. Equations in Table 1-3 are applicable for calculation of reference 5% offset yield,  $P$ , for dowel type fastener connections where the main member has a hollow cross section and the side member(s) is/are of solid cross section. The equations in Tables 1-2 and 1-3 are based on the assumption that the dowel penetrates both walls of the hollow member and the thickness of each wall through which the dowel penetrates,  $t_{ws}$  or  $t_{wm}$ , is the same. Connection yield modes are depicted in Tables 1-2 and 1-3. Example applications of equations in Table 1-3 are provided in Part 3 of this report.

## 1.6 Dowel Bearing Strength and Fastener Bending Yield Strength

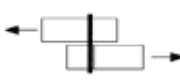
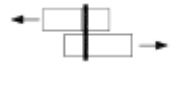
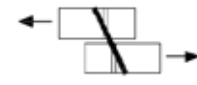
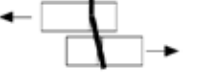
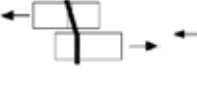
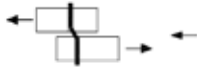
Dowel bearing strengths,  $F_e$ , and fastener bending yield strengths,  $F_{yb}$ , are provided in Appendix A.

### 1.7 Notation

$D$  = dowel diameter, in.  
 $D_s$  = dowel diameter at maximum stress in side member, in.  
 $D_m$  = dowel diameter at maximum stress in main member, in.  
 $D_r$  = fastener root diameter, in.  
 $E$  = length of tapered dowel tip, in.  
 $F_e$  = dowel bearing strength, psi  
 $F_{e\parallel}$  = dowel bearing strength parallel to grain in wood, psi  
 $F_{e\perp}$  = dowel bearing strength perpendicular to grain in wood, psi  
 $F_{es}$  = side member dowel bearing strength, psi  
 $F_{em}$  = main member dowel bearing strength, psi  
 $F_{tu}$  = tensile ultimate strength, psi  
 $F_u$  = specified minimum tensile strength, psi  
 $F_y$  = specified minimum tensile yield strength, psi  
 $F_{yb}$  = specified minimum dowel bending yield strength of fastener, psi  
 $G$  = specific gravity  
 $g$  = gap between members, in.  
 $L_s$  = side member dowel-bearing length, in.  
 $L_m$  = main member dowel-bearing length, in.

$M_{bm}$  = main member dowel bearing maximum moment, in.-lbs  
 $M_{bs}$  = side member dowel bearing maximum moment, in.-lbs  
 $M_{ds}$  = side member dowel moment resistance,  $F_{yb}(D_s^3/6)$ , in.-lbs  
 $M_{dm}$  = main member dowel moment resistance,  $F_{yb}(D_m^3/6)$ , in.-lbs  
 $M_s$  = maximum moment developed in side member at  $x_s$ , in.-lbs  
 $M_m$  = maximum moment developed in main member at  $x$ , in.-lbs  
 $P$  = reference 5% offset yield lateral connection value for a single fastener connection, lbs  
 $p$  = dowel penetration, in.  
 $q_s$  = side member dowel-bearing resistance,  $F_{es}D$ , lbs./in.  
 $q_m$  = main member dowel-bearing resistance,  $F_{em}D$ , lbs./in.  
 $t$  = thickness, in.  
 $t_{ws}$  = wall thickness in hollow side member, in.  
 $t_{wm}$  = wall thickness in hollow main member, in.  
 $v_s$  = length of void space along the axis of the dowel in hollow side member, in.  
 $v_m$  = length of void space along the axis of the dowel in hollow main member, in.  
 $x$  = distance from shear plane to maximum moment, in.  
 $R_d$  = NDS reduction term for dowel type fasteners  
 $Z$  = reference lateral design value for a single fastener connection, lbs  
 $Z'$  = adjusted lateral design value for a single fastener connection, lbs

**Table 1-1 General Dowel Equations for Solid Cross Section Members<sup>2</sup>**

Yield Mode	Single Shear	Double Shear	Description	
I <sub>m</sub>	$P = q_m L_m$	$P = q_m L_m$		
I <sub>s</sub>	$P = q_s L_s$	$P = 2 q_s L_s$		
II-IV	$P = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$	$P = \frac{-B + \sqrt{B^2 - 4AC}}{A}$	General equation for member bearing and dowel yielding	
<b>Inputs A, B, &amp; C for Yield Modes II-IV</b>				
II <sup>1</sup>	$A = \frac{l}{4q_s} + \frac{l}{4q_m}$	$B = \frac{L_s}{2} + g + \frac{L_m}{2}$	$C = -\frac{q_s L_s^2}{4} - \frac{q_m L_m^2}{4}$	
III <sub>m</sub> <sup>1</sup>	$A = \frac{l}{2q_s} + \frac{l}{4q_m}$	$B = g + \frac{L_m}{2}$	$C = -M_s - \frac{q_m L_m^2}{4}$	
III <sub>s</sub>	$A = \frac{l}{4q_s} + \frac{l}{2q_m}$	$B = \frac{L_s}{2} + g$	$C = -\frac{q_s L_s^2}{4} - M_m$	
IV	$A = \frac{l}{2q_s} + \frac{l}{2q_m}$	$B = g$	$C = -M_s - M_m$	

<sup>1</sup>Yield Modes II and III<sub>m</sub> do not apply for double shear connections.

<sup>2</sup>See Section 1.6 for notation.

**Table 1-2 General Dowel Equations for Solid Cross-Section Main Member and Hollow Cross Section Side Member(s)<sup>2</sup>**

Yield Mode	Single Shear	Double Shear	Description	
I <sub>m</sub>	$P = q_m L_m$	$P = q_m L_m$		
I <sub>s</sub>	$P = 2q_s t_{ws}$	$P = 4q_s t_{ws}$		
II-IV	$P = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$	$P = \frac{-B + \sqrt{B^2 - 4AC}}{A}$	General equation for member bearing and dowel yielding	
<b>Inputs A, B, &amp; C for Yield Modes II-IV</b>				
II <sup>1</sup>	$A = \frac{1}{4q_s} + \frac{1}{4q_m}$	$B = t_{ws} + v_s + g + \frac{L_m}{2}$	$C = -q_s t_{ws} (t_{ws} + v_s) - \frac{q_m L_m^2}{4}$	
III <sub>m</sub> <sup>1</sup>	$A = \frac{1}{2q_s} + \frac{1}{4q_m}$	$B = g + \frac{L_m}{2}$	$C = -M_s - \frac{q_m L_m^2}{4}$	
III <sub>s</sub>	$A = \frac{1}{4q_s} + \frac{1}{2q_m}$	$B = t_{ws} + v_s + g$	$C = -q_s t_{ws} (t_{ws} + v_s) - M_m$	
IV	$A = \frac{1}{2q_s} + \frac{1}{2q_m}$	$B = g$	$C = -M_s - M_m$	

<sup>1</sup>Yield Modes II and III<sub>m</sub> do not apply for double shear connections.<sup>2</sup>See Section 1.6 for notation.



**Table 1-3 General Dowel Equations for Hollow Cross Section Main Member and Solid Cross Section Side Member(s)<sup>2</sup>**

Yield Mode	Single Shear	Double Shear	Description	
I <sub>m</sub>	$P = 2q_m t_{wm}$	$P = 2q_m t_{wm}$		
I <sub>s</sub>	$P = q_s L_s$	$P = 2q_s L_s$		
II-IV	$P = \frac{-B + \sqrt{B^2 - 4AC}}{2A}$	$P = \frac{-B + \sqrt{B^2 - 4AC}}{A}$	General equation for member bearing and dowel yielding	
<b>Inputs A, B, &amp; C for Yield Modes II-IV</b>				
II <sup>1</sup>	$A = \frac{l}{4q_s} + \frac{l}{4q_m}$	$B = t_{wm} + v_m + g + \frac{L_s}{2}$	$C = -q_m t_{wm} (t_{wm} + v_m) - \frac{q_s L_s^2}{4}$	
III <sub>m</sub> <sup>1</sup>	$A = \frac{l}{2q_s} + \frac{l}{4q_m}$	$B = g + t_{wm} + v_m$	$C = -M_s - q_m t_{wm} (t_{wm} + v_m)$	
III <sub>s</sub>	$A = \frac{l}{4q_s} + \frac{l}{2q_m}$	$B = \frac{L_s}{2} + g$	$C = -\frac{q_s L_s^2}{4} - M_m$	
IV	$A = \frac{l}{2q_s} + \frac{l}{2q_m}$	$B = g$	$C = -M_s - M_m$	

<sup>1</sup>Yield Modes II and III<sub>m</sub> do not apply for double shear connections.

<sup>2</sup>See Section 1.6 for notation.

## PART 2. Equation Derivation

### 2.1 Introduction

The yield model used to develop the general dowel equations considers effects of dowel moment resistance and dowel bearing resistance on a connection's lateral strength. Based on the European Yield Model (Soltis 1991), connection strength is assumed to be reached when: (1) compressive strength of the member beneath the dowel is exceeded; or (2) one or more plastic hinges forms in the dowel. Behavior of the connection is assumed to be in accordance with yield modes depicted in Tables 1-1, 1-2 and 1-3. Dowel loading is assumed to be uniformly distributed and perpendicular to the axis of the dowel (e.g. ideally plastic deformation). Each yield mode addresses a specific loading condition on the dowel such that the dowel will remain in static equilibrium. General dowel equations can be obtained by considering equilibrium of forces within a connection exhibiting behavior in accordance with yield modes I – IV as shown in Tables 1-1, 1-2 and 1-3, and Figures 2-1 and 2-2. Applying this concept, a free-body diagram for each yield mode can be drawn, and principles of statics can be used to develop the general dowel equations.

#### 2.1.1 End Fixity and Friction

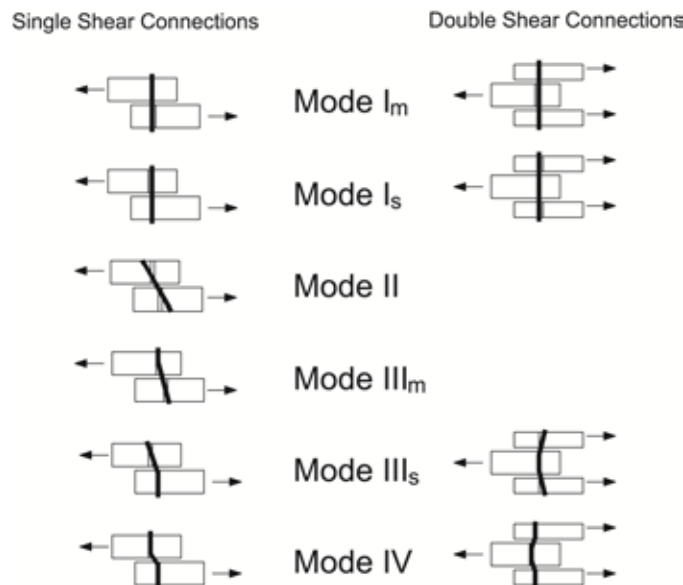
Potential increased strength due to effects of end fixity, tension forces in the fastener, and friction

between members are conservatively ignored. End fixity is the resistance to rotation provided at the end(s) of the dowel, such as under a nail head or bolt nut and washer. The strength contribution of end fixity is difficult to predict and may be dependent on several factors including load level, fastener type and installation, washer or fastener head size (e.g. nail or screw head size), tensile forces that may develop in the fastener under lateral loading, and amount of dimensional change in connected members. Frictional resistance to slipping of connection members is not typically addressed in wood connection design because the amount of frictional force is difficult to predict and in many instances may not exist as wood shrinks or the connection relaxes.

#### 2.1.2 Predicted Yield Mode

Predicted yield modes for single fastener connections are in agreement with yield modes observed from connection tests where end and edge distance, member cross section, and fastener size are adequate to ensure development of yielding by precluding occurrence of premature wood failures such as those due to splitting, formation of a wood shear plug, fastener failure or fastener withdrawal. These and other failure limit states are not specifically addressed by the yield mode equations.

**Figure 2-1 Connection Yield Modes – Solid Cross Section Members Section Members**



“m” denotes main member, “s” denotes side member

**2.2 Equation Derivation for Connections with Members of Solid Cross Section**

**2.2.1 Mode I**

Yield modes  $I_m$  and  $I_s$  model connections limited by uniform bearing in the main and side member(s). Figure 2-3, Case A, shows that the maximum value for shear,  $P$ , is determined by the following equations:

$$P = q_m L_m \tag{2}$$

$$P = q_s L_s \tag{3}$$

Similarly, considering the geometry of a double shear connection, the maximum value for shear,  $P$ , is determined by:

$$P = q_m L_m \tag{2}$$

$$P = 2q_s L_s \tag{4}$$

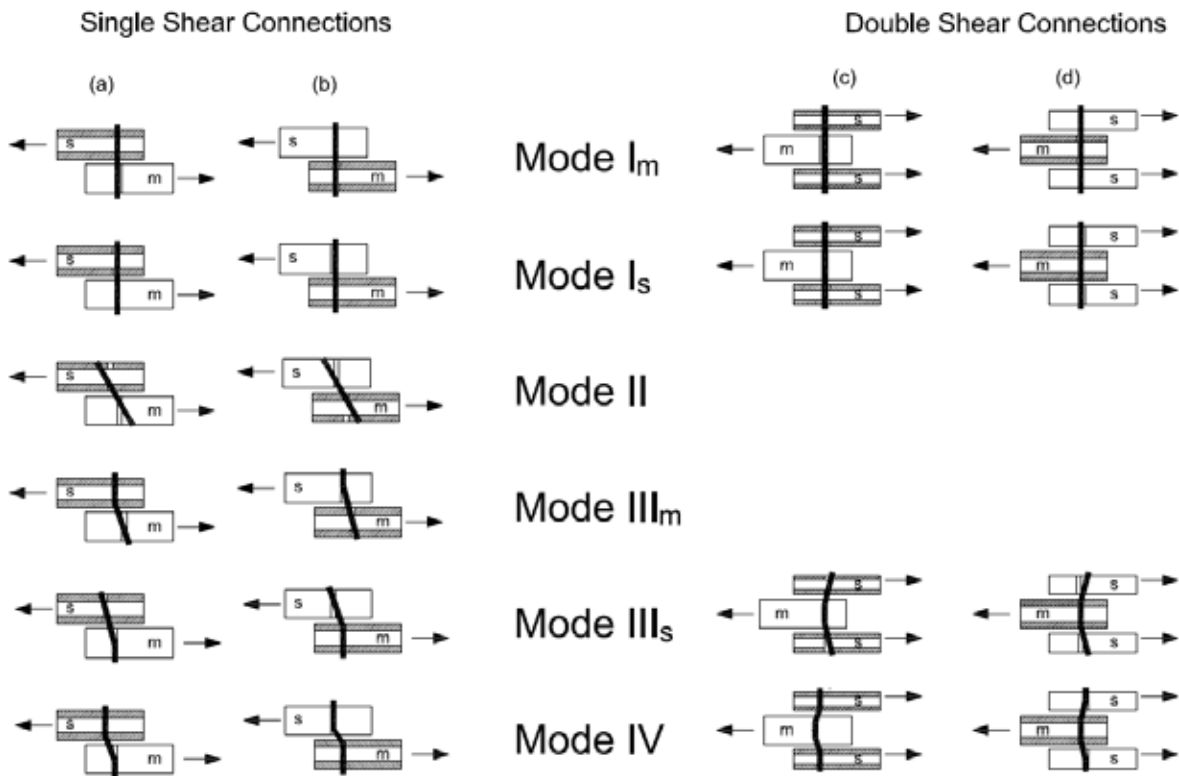
**2.2.2 Modes II-IV**

For modes II-IV, considering conditions where limiting or maximum moments act on the dowel

simplifies equation derivation. These basic conditions are shown in Figure 2-3. In Case B, maximum moment is based on dowel bearing. In Case C, maximum moment is based on dowel bending. Maximum moments for both cases occur at points of zero shear. Maximum moment due to dowel bearing, shown in Case B of Figure 2-3, represents a load condition where the dowel's moment capacity is sufficiently large to prevent yielding of the dowel (dowel bending). For calculation purposes, let  $q$  define member bearing resistance (lb/in.) ( $q = F_c D_b$ ),  $L$ , define member bearing length (in.), and let  $x$  represent the location of zero shear ( $x = L - 2a$ ). The resulting maximum moment due to dowel bearing,  $M_b$ , equals  $qa^2$ . Recognizing that  $a = (L - x)/2$  and  $x = P/q$  and substituting results in  $M_b = (q/4)(L - P/q)^2$  for single shear connections.

Subscripts  $s$  and  $m$  indicate side and main member in the following equations for maximum moment due to dowel bearing:

**Figure 2-2 Connection Yield Modes – Solid and Hollow Cross Section Members**



“m” denotes main member, “s” denotes side member

Single Shear:

$$M_{bs} = \frac{q_s}{4} \left( L_s - \frac{P}{q_s} \right)^2 \quad (5)$$

$$M_{bm} = \frac{q_m}{4} \left( L_m - \frac{P}{q_m} \right)^2 \quad (6)$$

Double Shear:

$$M_{bs} = \frac{q_s}{4} \left( L_s - \frac{P}{2q_s} \right)^2 \quad (7)$$

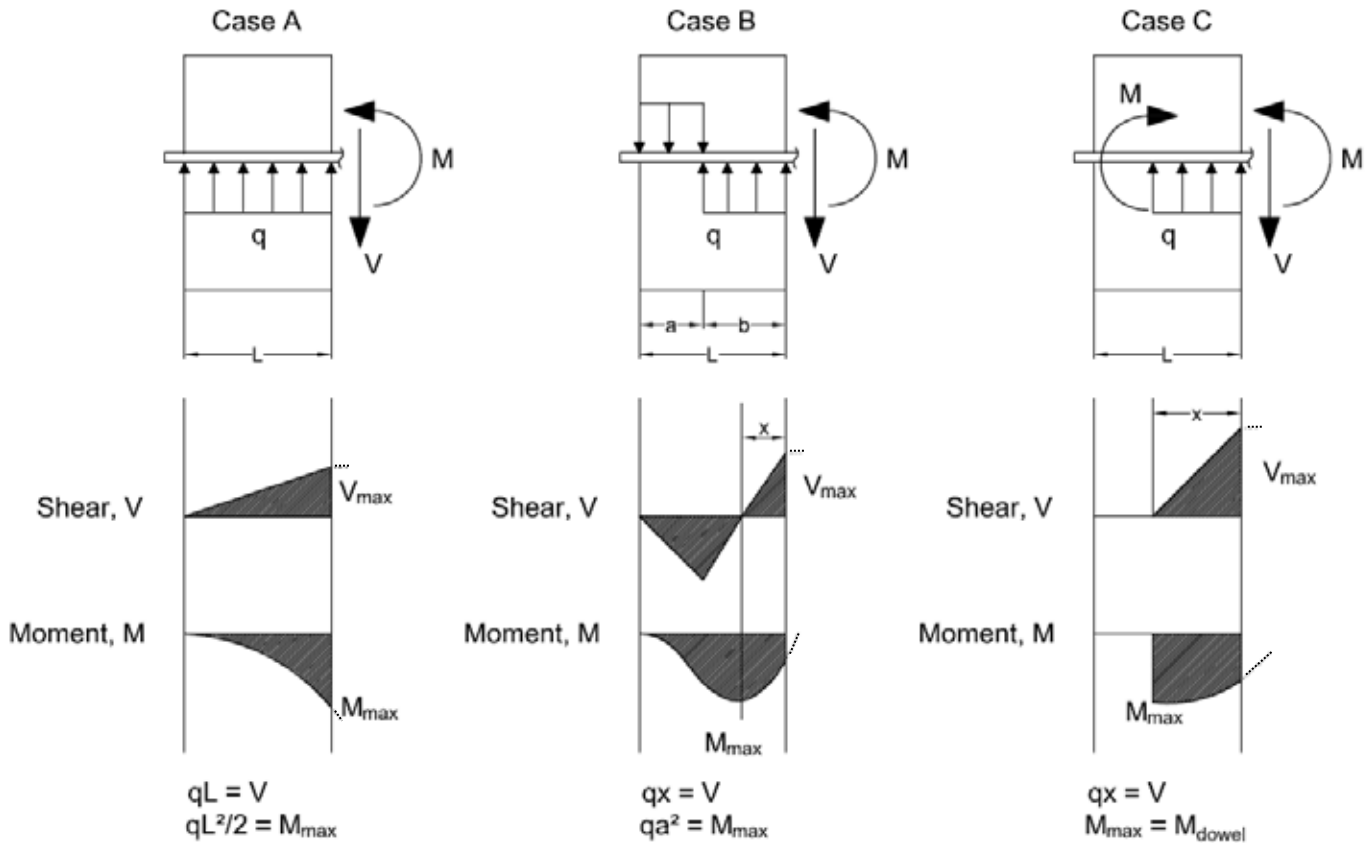
Maximum moment due to dowel bending is shown in Case C of Figure 2-3. In this case, maximum moment is limited to the moment provided by the dowel in bending which is represented by a concentrated moment acting at the point of zero shear ( $x = P/q$ ). Moment resistance of the dowel, assuming ideally plastic behavior, is expressed as follows:

$$M_{ds} = \frac{F_{yb} D_s^3}{6} \quad (8)$$

$$M_{dm} = \frac{F_{yb} D_m^3}{6} \quad (9)$$

Assumed solid section loading conditions for each of the yield modes I-IV is provided in Figure 2-4. Each mode consists of an interaction of dowel bearing and dowel bending as shown in Figure 2-3. Considering equilibrium of the dowel for each particular yield mode (by summing moments about a fixed point on the dowel), and using relationships for maximum moment defined above, characteristic equations for the maximum load,  $P$ , can be determined as follows:

**Figure 2-3 General Conditions of Dowel Loading – Solid Members**



Single Shear:

$$P^2 \left( \frac{1}{2q_s} + \frac{1}{2q_m} \right) + P g - (M_s + M_m) = 0 \quad (10)$$

Double Shear:

$$\frac{P^2}{4} \left( \frac{1}{2q_s} + \frac{1}{2q_m} \right) + \frac{P g}{2} - (M_s + M_m) = 0 \quad (11)$$

where:

$M_s$  = maximum moment developed in the side member at  $x_s$

$M_m$  = maximum moment developed in the main member at  $x_m$

$g$  = gap distance (assumed to be equal for double shear connections)

Variables  $M_s$  and  $M_m$  represent values of maximum moment due to dowel bearing or dowel bending depending on the mode being considered. An example single shear connection with assumed loading for yield mode II is provided in Figure 2-5. Shear and moment diagrams are also provided. As shown in Figure 2-5, maximum moments,  $M_s$  and  $M_m$  for mode II occur at distance  $x_s$  and  $x_m$  from connected faces, and are based on moments  $M_{bs}$  and  $M_{bm}$  due to dowel bearing.

Derivation of the general dowel equations assumes that critical stresses in the dowel occur at locations of maximum induced moment. This is appropriate for dowels having a constant diameter in the side and main member. Dowel diameters in the side and main member, however, do not need to be equal. For connections where dowel diameter is not constant within a member, it is conservative to assume that the least diameter occurs at the location of maximum moment. Alternatively, critical stress in the dowel can be determined by considering the applicable moment and dowel section properties along the length of the dowel.

Yield modes II and III<sub>m</sub> are not possible for double shear connections, as the assumed symmetry of double shear connections does not permit these modes to occur.

### 2.2.3 Mode II

Yield Mode II models a connection limited by dowel bearing in the side and main members. The maximum induced moment was previously determined as follows:

Single Shear:

$$M_{bs} = \frac{q_s}{4} \left( L_s - \frac{P}{q_s} \right)^2 \quad (5)$$

$$M_{bm} = \frac{q_m}{4} \left( L_m - \frac{P}{q_m} \right)^2 \quad (6)$$

Substituting  $M_{bs}$  and  $M_{bm}$  into the characteristic equation for  $M_s$  and  $M_m$  results in the following quadratic equation expressed in terms of known properties for dowel-bearing resistance ( $q_s$  and  $q_m$ ), dowel-bearing length ( $L_s$  and  $L_m$ ), and the gap between members ( $g$ ), and a single unknown variable,  $P$ .

Single Shear:

$$P^2 \left( \frac{1}{4q_s} + \frac{1}{4q_m} \right) + P \left( \frac{L_s}{2} + g + \frac{L_m}{2} \right) - \left( \frac{q_s L_s^2}{4} + \frac{q_m L_m^2}{4} \right) = 0 \quad (12)$$

### 2.2.4 Mode III<sub>m</sub>

Mode III<sub>m</sub> models a connection limited by dowel bearing in the main member and dowel bending in the side member. The maximum load,  $P$ , is determined by solving the characteristic equation for  $P$ :

Single Shear:

$$M_{bm} = \frac{q_m}{4} \left( L_m - \frac{P}{q_m} \right)^2 \quad (13)$$

and:

$M_{ds}$  = side member dowel moment resistance

Substituting  $M_{ds}$  and  $M_{bm}$  into the characteristic equation for  $M_s$  and  $M_m$  results in the following quadratic equation expressed in terms of known properties for dowel-bearing resistance ( $q_s$  and  $q_m$ ), dowel-bearing length ( $L_m$  and  $L_{ds}$ ), and the gap between members ( $g$ ), and a single unknown variable,  $P$ .

Single Shear:

$$P^2 \left( \frac{1}{2q_s} + \frac{1}{4q_m} \right) + P \left( g + \frac{L_m}{2} \right) - \left( M_{ds} + \frac{q_m L_m^2}{4} \right) = 0 \quad (14)$$

### 2.2.5 Mode III<sub>s</sub>

Mode III<sub>s</sub> models a connection limited by dowel bearing in the side member(s) and dowel bending in the main member. The maximum load,  $P$ , is determined by solving the characteristic equation for  $P$ .

Single Shear:

$$M_{bs} = \frac{q_s}{4} \left( L_s - \frac{P}{q_s} \right)^2 \quad (5)$$

and:

$M_m$  = dowel bending moment in the main member

Double Shear:

$$M_{bs} = \frac{q_s}{4} \left( L_s - \frac{P}{2q_s} \right)^2 \quad (7)$$

and:

$M_{dm}$  = main member dowel moment resistance

Substituting  $M_{bs}$  and  $M_{dm}$  into the characteristic equations (equations 10 and 11) for  $M_s$  and  $M_m$  results in the following quadratic equation(s) expressed in terms of known properties of  $q_s$ ,  $q_m$ ,  $L_s$ ,  $M_m$ , and  $g$ , and a single unknown variable,  $P$ .

Single Shear:

$$P^2 \left( \frac{1}{4q_s} + \frac{1}{2q_m} \right) + P \left( \frac{L_s}{2} + g \right) - \left( \frac{q_s L_s^2}{4} + M_{dm} \right) = 0 \quad (15)$$

Double Shear:

$$\frac{P^2}{4} \left( \frac{1}{4q_s} + \frac{1}{2q_m} \right) + \frac{P}{2} \left( \frac{L_s}{2} + g \right) - \left( \frac{q_s L_s^2}{4} + M_m \right) = 0 \quad (16)$$

### 2.2.6 Mode IV

Mode IV models a connection limited by dowel bending in the main and side member(s). The maximum load,  $P$ , is determined by solving the characteristic equation for  $P$ :

where:

$M_{ds}$  = side member dowel moment resistance

$M_{dm}$  = main member dowel moment resistance

Substituting  $M_{ds}$  and  $M_{dm}$  into the characteristic equations (equations 10 and 11) for  $M_s$  and  $M_m$  results in the following quadratic equations expressed in terms of known properties for dowel-bearing resistance ( $q_s$  and  $q_m$ ), dowel-bearing length ( $M_s$  and  $M_m$ ), and the gap between members ( $g$ ), and a single unknown variable,  $P$ .

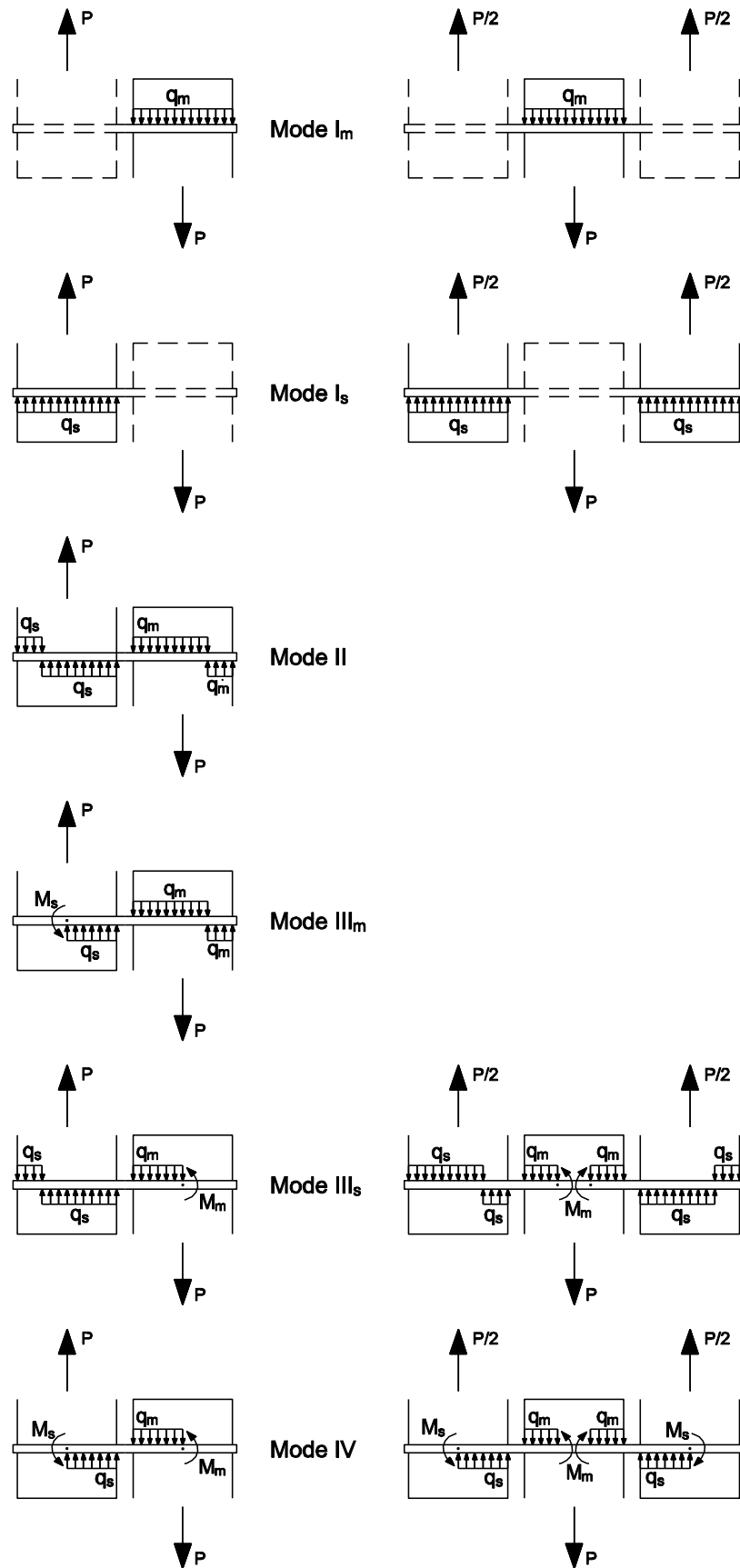
Single Shear:

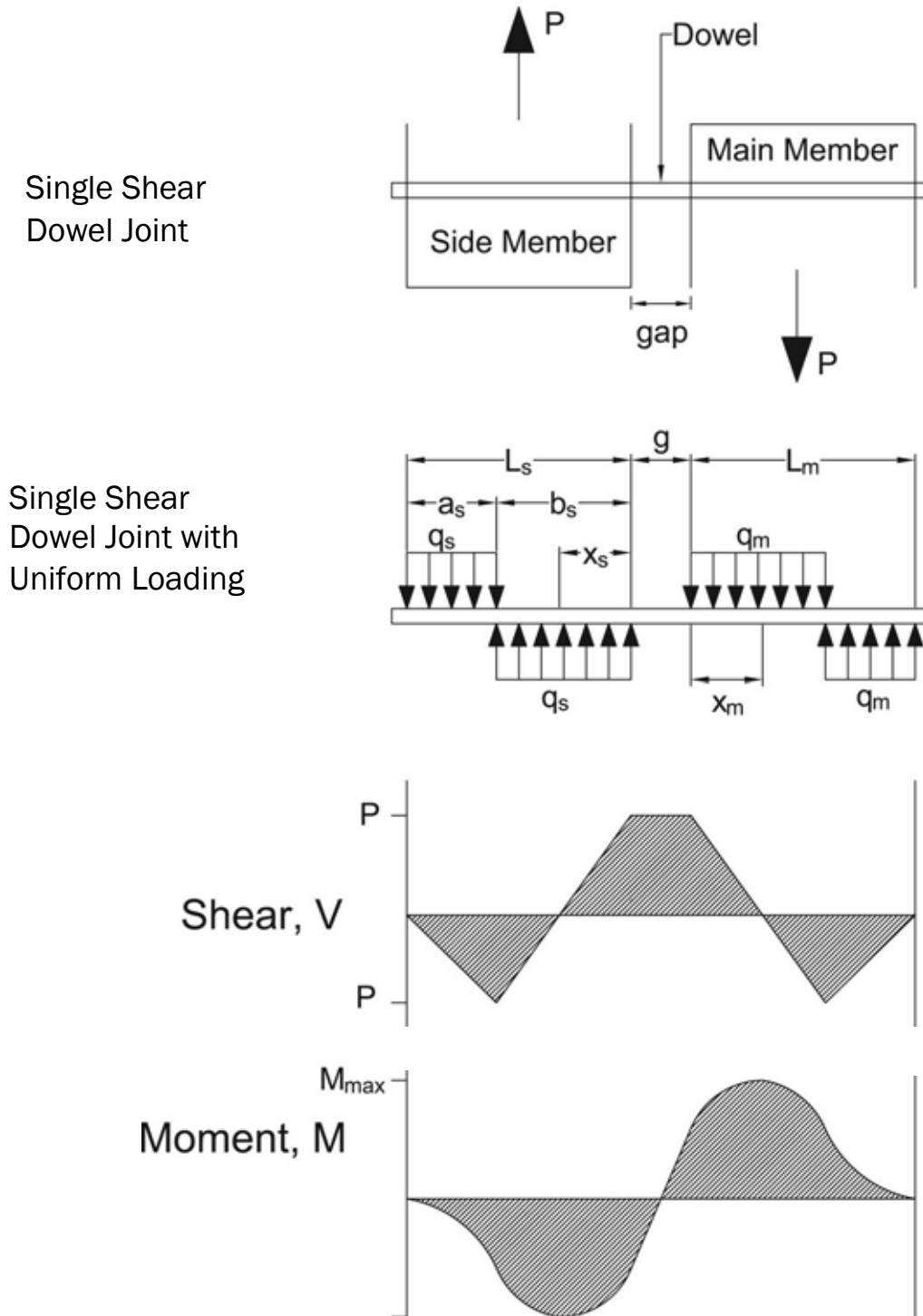
$$P^2 \left( \frac{1}{2q_s} + \frac{1}{2q_m} \right) + P g - (M_{ds} + M_{dm}) = 0 \quad (17)$$

Double Shear:

$$\frac{P^2}{4} \left( \frac{1}{2q_s} + \frac{1}{2q_m} \right) + \frac{P g}{2} - (M_{ds} + M_{dm}) = 0 \quad (18)$$

**Figure 2-4 Connection Yield Modes Assumed Loading – Solid Members**



**Figure 2-5 Single Shear Connection - Mode II**




### 2.3 Tapered Tip Equation Derivation

Equations considering effect of a tapered fastener tip are based on connections composed entirely of solid-cross section members but with bearing resistance in the penetrated member based on a combination of full diameter bearing and reduced diameter bearing of the tapered fastener tip.

#### 2.3.1 Mode I

The bearing resistance of the fastener with a tapered tip can be calculated as the combination of bearing resistance provided by the full diameter portion of the fastener plus the bearing resistance provided by the tapered tip which is modeled as having diameter vary linearly from full diameter to a point over the length of the tapered tip, E. Maximum load P is determined by:

$$P = q_m(p - E) + \frac{1}{2}q_m(E) \quad (19)$$

The Mode I<sub>m</sub> equation can be further simplified to:

$$P = q_m \left( p - \frac{E}{2} \right) \quad (20)$$

For single shear connections, bearing in the side member is based on diameter, D. Maximum load P is determined by the characteristic equation:

$$P = q_s L_s \quad (3)$$

Considering the geometry of a double shear connection, where the tip is in the side member, maximum load, P, is determined by:

$$P = q_m L_m \quad (2)$$

$$P = q_s \left( 2L_s - \frac{E}{2} \right) \quad (21)$$

#### 2.3.2 Modes II-IV

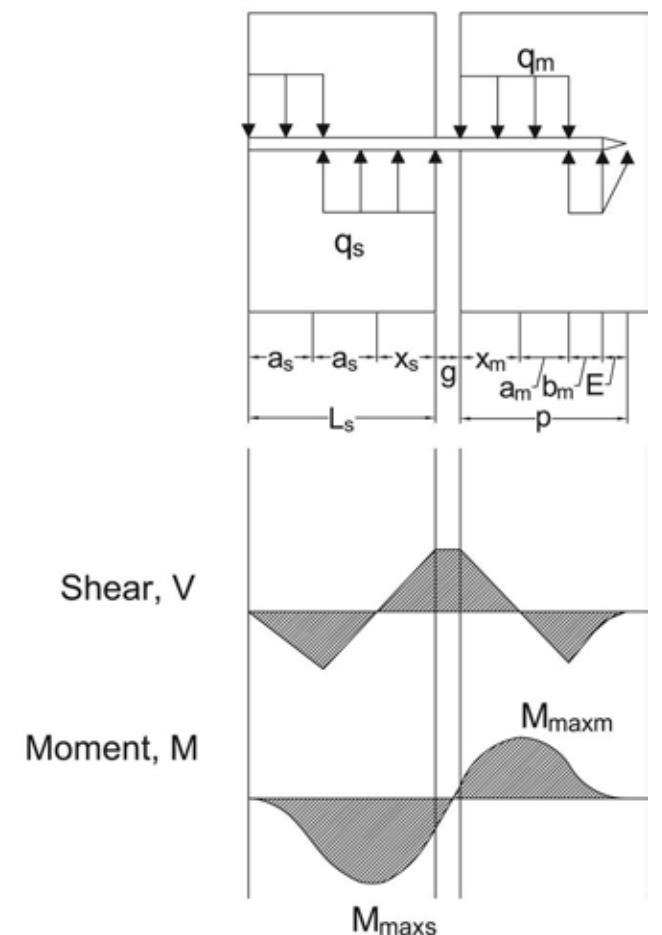
Figure 2-6 illustrates a general dowel bearing scenario with a tapered tip connector. For calculation purposes, let q define member bearing resistance (lb/in.) determined by the equation  $q = F_e D_b$ , p define penetration length (in), E define the length of taper (in), and let x represent the location of zero shear ( $x = p - a - b - E$ ). The resulting maximum moment due to dowel bearing, M<sub>b</sub>, equals  $qa^2 + qE^2/24$ . Recognizing that  $a_m = p/2 - x_m/2 - E/4$  and  $x = P/q$  results in the following equations for maximum moment due to dowel bearing:

$$M_{bs} = \frac{P^2}{4q_s} - \frac{PL_s}{2} + \frac{q_s L_s^2}{4} \quad (22)$$

$$M_{bm} = \frac{P^2}{4q_m} - P \left( \frac{p}{2} - \frac{E}{4} \right) + q_m \left( \frac{p^2}{4} - \frac{pE}{4} + \frac{5E^2}{48} \right) \quad (23)$$

The maximum moment due to dowel bending is limited to the moment provided by the dowel in bending which is represented by a concentrated moment acting at the point of zero shear ( $x = P/q$ ). Moment resistance of the dowel, M<sub>ds</sub> and M<sub>dm</sub>, assuming ideally plastic behavior, is expressed in equations 8 and 9.

**Figure 2-6 Dowel Bearing with Tapered Tip Connector**



Substitution of values of maximum bearing resistance and maximum moment resistance as applicable for the yield mode into characteristic equation for single shear (equation 10) and for double shear (equation 11) result in equations the maximum load, P, as follows:

**Mode II**

Single Shear:

$$P^2 \left( \frac{1}{4q_s} + \frac{1}{4q_m} \right) + P \left( \frac{L_s}{2} + g + \frac{p}{2} - \frac{E}{4} \right) - \left( \frac{q_s L_s^2}{4} + \frac{q_m p^2}{4} - \frac{q_m p E}{4} + \frac{q_m 5E^2}{48} \right) = 0 \quad (24)$$

**Mode III<sub>m</sub>**

Single Shear:

$$P^2 \left( \frac{1}{2q_s} + \frac{1}{4q_m} \right) + P \left( g + \frac{p}{2} - \frac{E}{4} \right) - \left( M_s + \frac{q_m p^2}{4} - \frac{q_m p E}{4} + \frac{q_m 5E^2}{48} \right) = 0 \quad (25)$$

**Mode III<sub>s</sub>**

Single Shear:

$$P^2 \left( \frac{1}{4q_s} + \frac{1}{2q_m} \right) + P \left( \frac{L_s}{2} + g \right) - \left( \frac{q_s L_s^2}{4} + M_{dm} \right) = 0 \quad (26)$$

Double Shear:

$$\frac{P^2}{4} \left( \frac{1}{4q_s} + \frac{1}{2q_m} \right) + \frac{P}{2} \left( \frac{L_s}{2} + g \right) - \left( \frac{q_s L_s^2}{4} + M_{dm} \right) = 0 \quad (27)$$

**Mode IV**

Single Shear:

$$P^2 \left( \frac{1}{2q_s} + \frac{1}{2q_m} \right) + P g - (M_{ds} + M_{dm}) = 0 \quad (28)$$

Double Shear:

$$\frac{P^2}{4} \left( \frac{1}{2q_s} + \frac{1}{2q_m} \right) + \frac{P g}{2} - (M_{ds} + M_{dm}) = 0 \quad (29)$$

For double shear where side member penetrations are unequal,  $L_s$  shall be taken as the minimum bearing length in either of the two side members.

**2.3.3 NDS Assumption**

For connectors with a tapered tip, the NDS specifies that bearing length is permitted to be calculated as  $L = p - E/2$ . Example 2.4 in Part 3 of this report shows that the NDS requirement closely approximates results from the more detailed evaluation of a tapered tip on bearing resistance in accordance with the foregoing derivations.

**2.4 Equation Derivation for Connections with Members of Hollow Cross Section**

Following the approach for connections composed entirely of members having solid cross sections, equations for connections composed of a combination of members having solid and hollow cross section can be developed as shown in Section 2.4 for yield modes depicted in Tables 1-2 and 1-3. Equations in Section 2.4 are based on assumed connection configurations composed entirely of hollow cross section members for both side and main members.

**2.4.1 Mode I**

Yield modes  $I_m$  and  $I_s$  model connections limited by uniform bearing in the main and side member(s). Figure 2-7, Case A, shows that the maximum value for shear,  $P$ , is determined by the following equations:

$$P = 2q_m t_{wm} \quad (30)$$

$$P = 2q_s t_{ws} \quad (31)$$

Similarly, considering the geometry of a double shear connection, the maximum value for shear,  $P$ , is determined by the equations:

$$P = 2q_m t_{wm} \quad (30)$$

$$P = 4q_s t_{ws} \quad (32)$$

**2.4.2 Modes II-IV**

For modes II-IV, considering conditions where limiting or maximum moments act on the dowel simplifies equation derivation (see Figure 2-7). In Case B, maximum moment is based on dowel bearing. In Case C, maximum moment is based on dowel bending. Maximum moments for both cases occur at points of zero shear. Parsons (2001) showed that for hollow cross section members, the controlling load cases show that all dowel yielding occurs in the wall next to the shear plane, therefore only these cases are considered in this document.

Maximum moment due to dowel bearing, shown in Case B of Figure 2-7, represents a load condition where the dowel is sufficiently large to prevent yielding of the dowel (dowel bending). For calculation purposes, let  $q$  define member bearing resistance (lb/in.) ( $q = F_e D_b$ ),  $t_w$  define member wall thickness (in.), and let  $x$  represent the location of zero shear ( $x = 2(t_w - a)$ ).

The resulting maximum moment due to dowel bearing,  $M_b$ , equals  $qa^2 + 2qav - qt_w v$ . Recognizing that  $a = t_w - x/2$  and  $x = P/q$ , and substituting, results in  $M_b = P^2/(4q) - P(t_w + v) + qt_w^2 + qt_w v$  in the member.

Subscripts *s* and *m* indicate side and main member in the following equations for maximum moment due to dowel bearing:

Single Shear:

$$M_{bs} = \frac{P^2}{4q_s} - P(t_{ws} + v_s) + q_s t_{ws}^2 + q_s t_{ws} v_s \quad (33)$$

$$M_{bm} = \frac{P^2}{4q_m} - P(t_{wm} + v_m) + q_m t_{wm}^2 + q_m t_{wm} v_m \quad (34)$$

Double Shear:

$$M_{bs} = \frac{P^2}{16q_s} - P\left(\frac{t_{ws}}{2} + \frac{v_s}{2}\right) + q_s t_{ws}^2 + q_s t_{ws} v_s \quad (35)$$

Maximum moment due to dowel bending is shown in Case C of Figure 2-7. In this case, maximum moment is limited to the moment provided by the dowel in bending which is represented by a concentrated moment acting at the point of zero shear ( $x = P/q$ ). Moment resistance of the dowel, assuming

ideally plastic behavior, is expressed in equations 8 and 9.

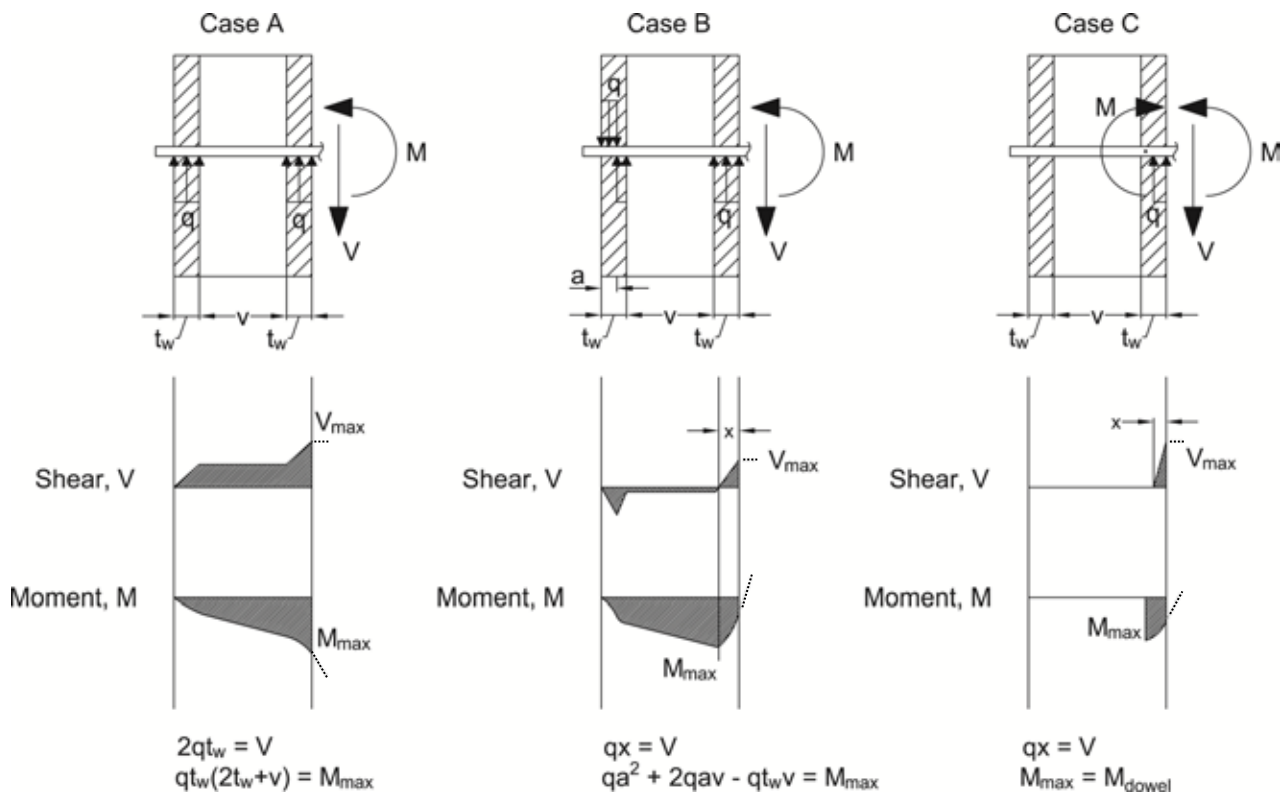
Following the method established in Section 2.2, the maximum load, *P*, can be determined using the same characteristic equations used for solid members (equations 10 and 11).

Similar to solid members, yield modes II and III<sub>m</sub> are not possible for double shear connections, as the assumed symmetry of double shear connections does not permit these modes to occur.

**2.4.3 Mode II**

Yield Mode II models a connection limited by dowel bearing in the side and main members. The maximum induced moment was previously determined (equations 33 and 34). Substituting *M*<sub>bs</sub> and *M*<sub>bm</sub> into the characteristic equation for *M*<sub>s</sub> and *M*<sub>m</sub> results in the following quadratic equation, expressed in terms of known properties for dowel-bearing resistance (*q*<sub>s</sub> and *q*<sub>m</sub>), dowel-bearing length (*L*<sub>s</sub> and *L*<sub>m</sub>), and the gap between members (*g*), and a single unknown variable, *P*.

**Figure 2-7 General Conditions of Dowel Loading – Hollow Cross Section Members**



Single Shear:

$$P^2 \left( \frac{1}{4q_s} + \frac{1}{4q_m} \right) + P(t_{ws} + v_s + g + t_{wm} + v_m) - (q_s t_{ws}^2 + q_s t_{ws} v_s + q_m t_{wm}^2 + q_m t_{wm} v_m) = 0 \quad (36)$$

### 2.4.4 Mode III<sub>m</sub>

Mode III<sub>m</sub> models a connection limited by dowel bearing in the main member and dowel bending in the side member. Dowel bending will occur in the member wall closest to the shear plane. Substituting  $M_{ds}$  (equation 8) and  $M_{bm}$  (equation 34) into the characteristic equation (equations 10 and 11) for  $M_s$  and  $M_m$  results in the following quadratic equation expressed in terms of known properties for dowel-bearing resistance ( $q_s$  and  $q_m$ ), dowel-bearing length ( $L_m$ ), dowel moment resistance ( $M_{ds}$ ) and the gap between members ( $g$ ), and a single unknown variable,  $P$ .

Single Shear:

$$P^2 \left( \frac{L}{2q_s} + \frac{L}{4q_m} \right) + P(g + t_{wm} + v_m) - (M_{ds} + q_m t_{wm}^2 + q_m t_{wm} v_m) = 0 \quad (37)$$

### 2.4.5 Mode III<sub>s</sub>

Mode III<sub>s</sub> models a connection limited by dowel bearing in the side member(s) and dowel bending in the main member. Substituting  $M_{bs}$  (equations 33 and 35) and  $M_{dm}$  (equation 9) into the characteristic equation for  $M_s$  and  $M_m$  results in the following quadratic equation(s) expressed in terms of known

properties for dowel-bearing resistance ( $q_s$  and  $q_m$ ), dowel-bearing length ( $L_s$ ), dowel moment resistance ( $M_{dm}$ ) and the gap between members ( $g$ ), and a single unknown variable,  $P$ .

Single Shear:

$$P^2 \left( \frac{L}{4q_s} + \frac{L}{2q_m} \right) + P(t_{ws} + v_s + g) - (q_s t_{ws}^2 + q_s t_{ws} v_s + M_{dm}) = 0 \quad (38)$$

Double Shear:

$$\frac{P^2}{4} \left( \frac{L}{4q_s} + \frac{L}{2q_m} \right) + \frac{P}{2}(t_{ws} + v_s + g) - (q_s t_{ws}^2 + q_s t_{ws} v_s + M_{dm}) = 0 \quad (39)$$

### 2.4.6 Mode IV

Mode IV models a connection limited by dowel bending in the main and side member(s). Substituting  $M_{ds}$  and  $M_{dm}$  (equations 8 and 9) into the characteristic equations for  $M_s$  and  $M_m$  yields the same quadratic equations developed for solid sections (equations 17 and 18).

An alternative, energy based derivation, producing the same results in different equation format, is provided in Parsons (2001).

## PART 3. Example Problems

### Example 3.1 Bolted Connection with Gap

**Problem Statement:** Determine the reference design value,  $Z$ , for a wood-to-wood single shear bolted connection. Compare values for gap distance,  $g$ , equal to 0 in.,  $\frac{1}{4}$  in., and  $\frac{1}{2}$  in. Connection end and edge distances are assumed to be in accordance with applicable NDS provisions.

**Given:** Both side and main member thicknesses are 1.5 in. Each connection uses a single  $\frac{1}{2}$  in. diameter bolt (SAE J429 Grade 1).

$$L_s = L_m = 1.5 \text{ in.}$$

$$D = 0.5 \text{ in.}$$

$$F_{yb} = 45,000 \text{ psi}$$

$$F_{ell} = 4800 \text{ psi, } F_{e\perp} = 2550 \text{ psi}$$

$$g = 0, 0.25, 0.5 \text{ in.}$$

Bearing length, in. & grain direction				$P_{5\%/R_d}$ , lbs				$Z^1$ , lbs
Main	Side	$I_m$	$I_s$	II	III <sub>m</sub>	III <sub>s</sub>	IV	
<b>Gap distance, <math>g = 0</math> in.</b>								
1- $\frac{1}{2}$ <sub>  </sub>	1- $\frac{1}{2}$ <sub>  </sub>	900	900	<u>414</u>	550	550	663	414
1- $\frac{1}{2}$ <sub>  </sub>	1- $\frac{1}{2}$ <sub>⊥</sub>	720	383	<u>250</u>	380	324	442	250
1- $\frac{1}{2}$ <sub>⊥</sub>	1- $\frac{1}{2}$ <sub>⊥</sub>	383	383	<u>176</u>	289	289	387	176
<b>Gap distance, <math>g = \frac{1}{4}</math> in.</b>								
1- $\frac{1}{2}$ <sub>  </sub>	1- $\frac{1}{2}$ <sub>  </sub>	900	900	<u>370</u>	482	482	576	370
1- $\frac{1}{2}$ <sub>  </sub>	1- $\frac{1}{2}$ <sub>⊥</sub>	720	383	<u>224</u>	341	284	393	224
1- $\frac{1}{2}$ <sub>⊥</sub>	1- $\frac{1}{2}$ <sub>⊥</sub>	383	383	<u>157</u>	258	258	349	157
<b>Gap distance, <math>g = \frac{1}{2}</math> in.</b>								
1- $\frac{1}{2}$ <sub>  </sub>	1- $\frac{1}{2}$ <sub>  </sub>	900	900	<u>333</u>	426	426	501	333
1- $\frac{1}{2}$ <sub>  </sub>	1- $\frac{1}{2}$ <sub>⊥</sub>	720	383	<u>202</u>	307	250	350	202
1- $\frac{1}{2}$ <sub>⊥</sub>	1- $\frac{1}{2}$ <sub>⊥</sub>	383	383	<u>142</u>	231	231	315	142

<sup>1</sup>Connection values have not been adjusted for end use conditions such as load duration, wet service, and temperature.  $Z$  is the minimum of  $P_{5\%/R_d}$  for each yield mode. The controlling yield mode value is underlined.

### Example 3.2 Lag Screw Connection

**Problem Statement:** Determine the reference design value for a wood-to-wood single shear lag screw connection. Compare values for different connection conditions described in Cases 1–4. The main member is loaded parallel to grain for all cases. Connection end and edge distances are assumed to be in accordance with applicable NDS provisions.

**Note:** This example illustrates the application of TR12 equations to account for varying bearing and moment resistances of lag screws in a wood-to-wood connection. A similar approach can be applied to

*other threaded fasteners such as bolts and wood screws by accounting for specific attributes (diameter, root diameter, dowel bending strength) of those fastener types.*

**Given Data:**

Parallel and perpendicular to grain dowel bearing strengths:

$$F_{ell} = 5600 \text{ psi, } F_{e\perp} = 3650 \text{ psi}$$

$$F_{yb} = 45,000 \text{ psi}$$

$$L_s = 1.5 \text{ in.}$$

$$L_m = 3 \text{ in.}$$

$$D = 0.375 \text{ in.,}$$

$$D_r = 0.265 \text{ in.}$$

$$g = 0 \text{ in.}$$

**Case 1 (Figure 3-2A):** Moment resistance, side member bearing, and main member bearing are based on fastener root diameter,  $D_r$ . *Note: These are the assumptions used to develop the NDS tabular values.*

Side Member:

$$q_s = F_{es}D_r$$

$$M_s = \frac{F_{yb}D_r^3}{6}$$

Main Member:

$$q_m = F_{em}D_r$$

$$M_m = \frac{F_{yb}D_r^3}{6}$$

**Case 2 (Figure 3-2B):** Moment resistance and side member bearing are based on fastener root diameter,  $D_r$ . Bearing resistance in the main member is based on  $D$ .

Side Member:

$$q_s = F_{es}D_r$$

$$M_s = \frac{F_{yb}D_r^3}{6}$$

Main Member:

$$q_m = F_{em}D$$

$$M_m = \frac{F_{yb}D_r^3}{6}$$

**Case 3 (Figure 3-2C):** Moment resistance is based on fastener root diameter,  $D_r$ , and bearing resistance is based on  $D$ .

Side Member:

$$q_s = F_{es}D$$

$$M_s = \frac{F_{yb}D_r^3}{6}$$

Main Member:

$$q_m = F_{em}D$$

$$M_m = \frac{F_{yb}D_r^3}{6}$$

**Case 4 (Figure 3-2D):** Fastener moment resistance and bearing resistance are based on diameter,  $D$  (e.g. connection with unthreaded shank extending deep into the main member). Additional calculations will determine the required length of unthreaded shank penetration into the main member.

*Note: Where unthreaded shank penetration into the main member is not adequate to enable development of the moment resistance of the fastener based on diameter  $D$ , the fastener root diameter,  $D_r$ , is the appropriate diameter for calculation of fastener moment resistance in the main member.*

Side Member:

$$q_s = F_{es}D$$

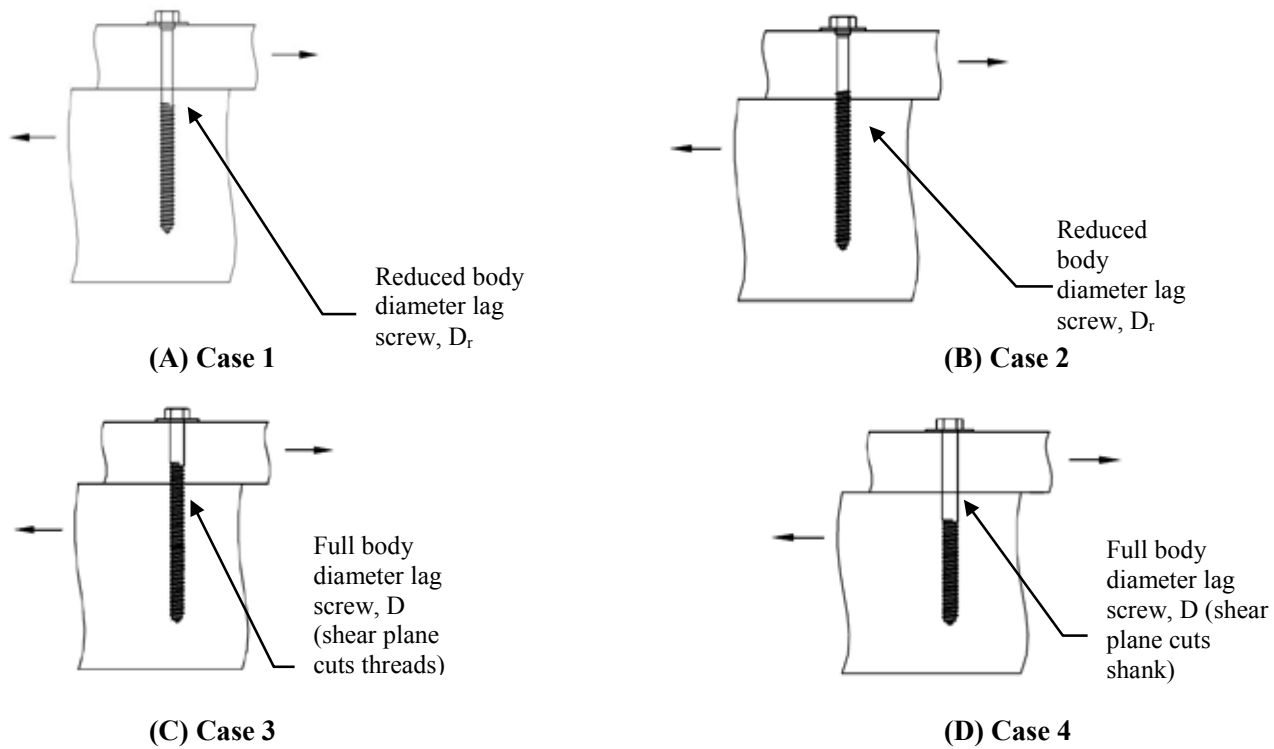
$$M_s = \frac{F_{yb}D^3}{6}$$

Main Member:

$$q_m = F_{em}D$$

$$M_m = \frac{F_{yb}D^3}{6}$$

**Figure 3-2 Wood-to-wood single shear lag screw comparison of different diameter and thread location conditions.**



Bearing length, in. & grain direction	$P_{5\%}/R_d$ , lbs						$Z^1$ , lbs
	$I_m$	$I_s$	II	III <sub>m</sub>	III <sub>s</sub>	IV	
<b>Case 1 (NDS Tabular Basis)<sup>2</sup></b>							
1- 1/2 <sub>  </sub>	1113	557	420	478	260	<u>201</u>	201
1- 1/2 <sub>⊥</sub>	890	290	304	353	153	<u>143</u>	143
<b>Case 2</b>							
1- 1/2 <sub>  </sub>	1575	557	548	629	275	<u>218</u>	218
1- 1/2 <sub>⊥</sub>	1260	290	400	457	160	<u>152</u>	152
<b>Case 3</b>							
1- 1/2 <sub>  </sub>	1575	788	595	671	357	<u>239</u>	239
1- 1/2 <sub>⊥</sub>	1260	411	431	495	207	<u>170</u>	170
<b>Case 4</b>							
1- 1/2 <sub>  </sub>	1575	788	595	697	406	<u>403</u>	403
1- 1/2 <sub>⊥</sub>	1260	411	431	513	<u>249</u>	286	286

<sup>1</sup>Connection values have not been adjusted for end use conditions such as load duration, wet service, and temperature.  $Z$  is the minimum of  $P_{5\%}/R_d$  for each yield mode. The controlling yield mode value is underlined.

<sup>2</sup>Case 1 represents assumptions used to develop NDS tabular values. Since the NDS equations utilize only a single diameter in calculation of reference design value, the least diameter (e.g.  $D_r$ ) is conservatively used to estimate moment and bearing resistances. In this example, Cases 2-4 demonstrate that more efficient designs can be achieved by considering the varying moment and bearing resistances of the fastener in lieu of the least diameter assumption.

### Example 3.3 Lag Screw Penetration

**Problem Statement:** Determine the minimum required length of shank penetration in the main member to ensure that fastener moment resistance in the main member is limited by diameter,  $D$ , of the shank rather than root diameter,  $D_r$ , at the threads. Connection end and edge distances are assumed to be in accordance with applicable NDS provisions.

**Note:** This example utilizes a lag screw connection to illustrate an approach for determining minimum required length of shank penetration to ensure fastener moment resistance in the main member is limited by diameter,  $D$ . A similar approach can be applied to other threaded fasteners such as bolts and wood screws by accounting for specific dimensions (diameter, root diameter, dowel bending strength) of those fasteners.

**Given:** The 3/8" diameter lag screw connection in Figure 3.3A has a reference design value of 403 lbs (Mode IV) based on diameter,  $D$  (see Case 4 of Example 3.2).

$$P = (\text{Reference design value})(R_d) \\ = (403 \text{ lbs})(3.2) = 1289.6 \text{ lbs}$$

$$q_m = F_{em}D \\ = (5600 \text{ psi})(0.375 \text{ in.}) = 2100 \text{ lb/in.}$$

Fastener moment resistance at shank:

$$D = \frac{F_{yb,5\%}D^3}{6} = 395.5 \text{ in.-lbs}$$

Fastener moment resistance at root:

$$D_r = \frac{F_{yb,5\%}D_r^3}{6} = 139.6 \text{ in.-lbs}$$

#### Solution:

The basic dowel loading condition for Mode IV is shown in Figure 3.3B. Since the reference value is based on Mode IV yield, the maximum moment is equal to the moment resistance of the dowel (e.g.  $M_{\max} = 395.5 \text{ in.-lbs}$  based on the moment resistance provided by shank diameter,  $D$ ). Zero shear and maximum moment occur at distance,  $x_m$ , from the shear plane:

$$x_m = \frac{P}{q_m} = \frac{1289.6}{2100} = 0.61 \text{ in.}$$

To estimate moment beyond  $x_m$  an alternate loading condition can be constructed for the dowel as shown in Figure 3.3C. The length of bearing needed to develop the maximum moment is calculated as:

$$2a = 2 \left( \frac{M_{\max}}{q_m} \right)^{0.5} = 2 \left( \frac{395.5}{2100} \right)^{0.5} = 0.87 \text{ in.}$$

The fastener moment diagram in the main member from  $0 \leq x_1 \leq 2a$  can be constructed from the alternate load condition in Figure 2.3C.

For  $0 < x_1 \leq a$ :

$$M = M_{\max} - \frac{q_m x_1^2}{2}$$

Substituting  $M_{\text{reduced}}$  for  $M$ , distance  $x_1$  from the point of maximum moment to the point of reduced moment can be calculated as:

$$x_1 = \left( \frac{2(M_{\max} - M_{\text{reduced}})}{q_m} \right)^{0.5}$$

Distance  $x_2$  from the shear plane to the point of reduced moment can be calculated as:

$$x_2 = \frac{P}{q_m} + \left( \frac{2(M_{\max} - M_{\text{reduced}})}{q_m} \right)^{0.5}$$

For  $a < x_1 \leq 2a$

$$M = M_{\max} - \frac{q_m a^2}{2} - \left( q_m a - \frac{q_m x_1}{2} \right) x_1$$

Substituting  $M_{\text{reduced}}$  for  $M$ , distance  $x_1$  from the point of maximum moment to the point of reduced moment can be calculated as:

$$x_1 = 2a - \left( \frac{2M_{\text{reduced}}}{q_m} \right)^{0.5}$$

Distance  $x_2$  from the shear plane to the point of reduced moment can be calculated as:

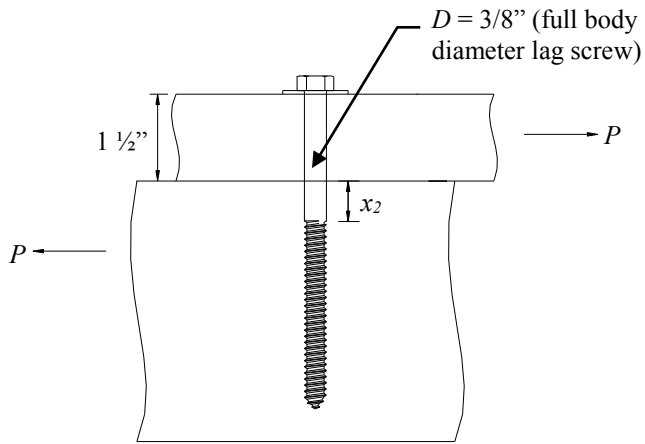
$$x_2 = \frac{P}{q_m} + 2a - \left( \frac{2M_{\text{reduced}}}{q_m} \right)^{0.5}$$

#### Minimum penetration length

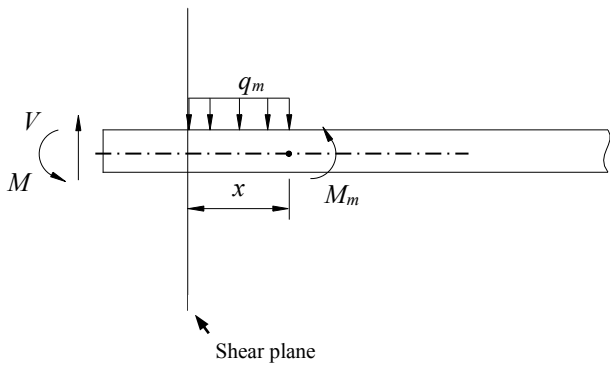
The value of  $M_{\text{reduced}}$  is taken as 139.6 in.-lbs which equals the moment resistance at the root diameter,  $D_r$ . For  $M_{\text{reduced}} = 139.6 \text{ in.-lbs}$ ,  $a < x_1 \leq 2a$  and the minimum required shank penetration,  $x_2$ , equals 1.12 in.



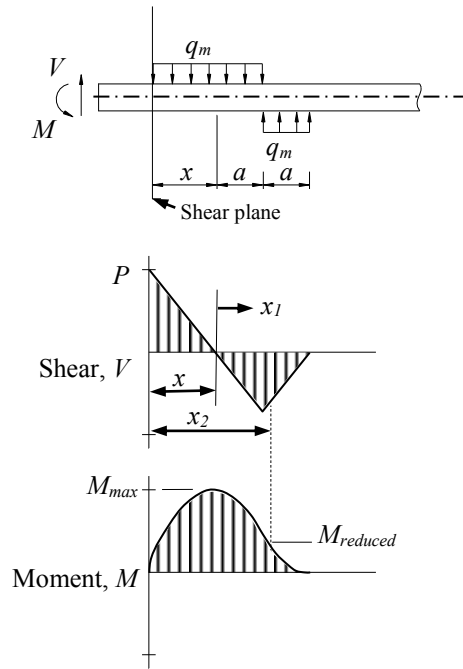
**Figure 3-3A**



**Figure 3-3B**



**Figure 3-3C**



### Example 3.4 Steel Side Member to Wood Main Member – Tapered Tip Effects

**Problem Statement:** Compare reference design values,  $Z$ , for a single shear nail connection at 12D, 10D, 8D, 6D, and 4D penetrations. Connection end and edge distances are assumed to be in accordance with applicable NDS provisions.

For “EYM tip derivation” case:

$$L_m = p$$

Use tapered tip equations 24 through 29

For “NDS” case:

$$L_m = p - E/2$$

Use general dowel equations in Table 1-1

**Given:** Dowel bearing strength is 4700 psi. Side member is 16 gage ASTM A653, Grade 33 steel. Nail diameter is 0.131" and is of adequate length to accommodate the given penetrations. Tapered tip length,  $E$ , is equal to  $2D$ . Main member is loaded parallel to grain.

$$L_s = 0.06 \text{ in.}$$

$$D = 0.131 \text{ in.}$$

$$E = 0.262 \text{ in.}$$

$$F_{em} = 4700 \text{ psi}$$

$$F_{es} = 61,850 \text{ psi}$$

$$F_{yb} = 100,000 \text{ psi}$$

$$g = 0 \text{ in.}$$

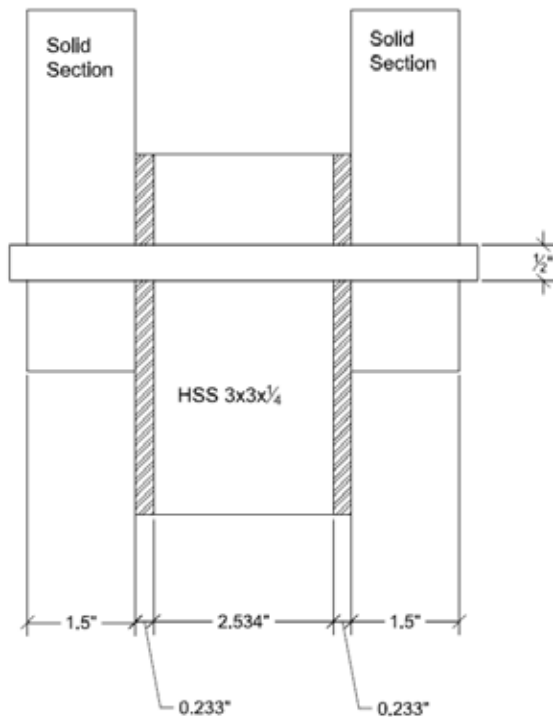
Penetration Depth (p)	EYM Tip derivation		NDS ( $L_m = p - E/2$ )	
	$Z^1$ (lbs)	Controlling Mode	$Z^1$ (lbs)	Controlling Mode
12D (1.57")	97	III <sub>s</sub>	97	III <sub>s</sub>
10D (1.31")	97	III <sub>s</sub>	97	III <sub>s</sub>
8D (1.05")	97	III <sub>s</sub>	97	III <sub>s</sub>
6D (0.79")	79	II	78	II

<sup>1</sup>Connection values have not been adjusted for end use conditions such as load duration, wet service, and temperature.  $Z$  is the minimum of  $P_{5\%}/R_d$  for each yield mode.

**Example 3.5 Bolted Connection through Solid and Hollow Members – Double Shear**

**Problem Statement:** Using equation in Table 1-3, determine the reference design value,  $Z$ , for a double shear bolted connection with a hollow section main member and solid cross section side members shown in Figure 3.5. Connection end and edge distances are assumed to be in accordance with applicable NDS provisions.

**Figure 3-5**



**Given:** Main member is an HSS 3x3x1/4, side members are 1.5" thick. A single 1/2" diameter SAE J429 Grade 1 bolt is used to make the connection.

$$t_{wm} = 0.233 \text{ in.}$$

$$v_m = 2.534 \text{ in.}$$

$$L_s = 1.5 \text{ in.}$$

$$D = 0.5 \text{ in.}$$

$$F_{em} = 87,000 \text{ psi}$$

$$F_{es} = 4800 \text{ psi}$$

$$F_{yb} = 45,000 \text{ psi}$$

$$g = 0 \text{ in.}$$

$$q_m = F_{em}D = 87,000 \text{ psi}(0.5 \text{ in.}) = 43,500 \text{ lb/in.}$$

$$q_s = F_{es}D = 4800 \text{ psi}(0.5 \text{ in.}) = 2400 \text{ lb/in.}$$

$$M_m = M_s = \frac{F_{yb}D^3}{6} = \frac{45,000 \text{ psi}(0.5 \text{ in.})^3}{6} = 937.5 \text{ in.-lb}$$

$P_{5\%}/R_d$ , lbs				$Z^1$ , lbs
$I_m$	$I_s$	$III_s$	IV	
5068	1800	<u>1413</u>	1825	1413

<sup>1</sup>Connection values have not been adjusted for end use conditions such as load duration, wet service, and temperature.  $Z$  is the minimum of  $P_{5\%}/R_d$  for each yield mode. The controlling yield mode value is underlined.

## **PART 4. References**

---

ANSI/AWC NDS-2012 *National Design Specification (NDS) for Wood Construction*. American Wood Council, Leesburg, VA, 2012.

Parsons, W.R. (2001). *Energy-based modeling of dowel-type connections in wood-plastic composite hollow sections*. Masters Thesis, Washington State University. Pullman WA.

Soltis, L.A. *European Yield Model for Wood Connections*. New York, NY: American Society of Civil Engineers, Proceedings of Structures Congress 1991, Indianapolis, IN 60-63; 1991.

## Appendix A. Dowel Bearing Strength and Fastener Bending Yield Strength

### A.1 Introduction

This Appendix provides dowel bearing strengths,  $F_e$ , and fastener bending yield strengths,  $F_{yb}$ , for use in calculation of reference lateral design values,  $Z$ , using Technical Report 12 (TR12) *General Dowel Equations for Calculating Lateral Connection Values*. Dowel bearing strengths and fastener bending yield strengths provided herein are identical to values established in the *National Design Specification® (NDS®) for Wood Construction* for materials included in connection design value tables [1]. Additional information on dowel bearing strengths and fastener bending yield strengths is provided including information on dowel bearing strengths of aluminum and stainless steel, and bending yield strengths for stainless steel nails.

### A.2 Notation

$D$	= dowel diameter, in.
$f'_c$	= specified concrete compressive strength, psi
$F_e$	= dowel bearing strength, psi
$F_{e\parallel}$	= dowel bearing strength parallel to grain in wood, psi
$F_{e\perp}$	= dowel bearing strength perpendicular to grain in wood, psi
$F_{tu}$	= tensile ultimate strength, psi
$F_u$	= specified minimum tensile strength, psi
$F_y$	= specified minimum tensile yield strength, psi
$F_{yb}$	= dowel bending yield strength of fastener, psi
$G$	= specific gravity of wood (oven dry weight and oven dry volume)
$G_{eqv}$	= test-based equivalent specific gravity of structural composite lumber
$t$	= thickness, in.
$Z$	= reference lateral design value for a single fastener connection, lb.
$Z'$	= adjusted lateral design value for a single fastener connection, lb.

### A.3 Dowel Bearing Strength

Dowel bearing strengths,  $F_e$ , for wood, steel, aluminum, and concrete used in calculation of lateral connection design values,  $Z$ , are summarized in Table A1.

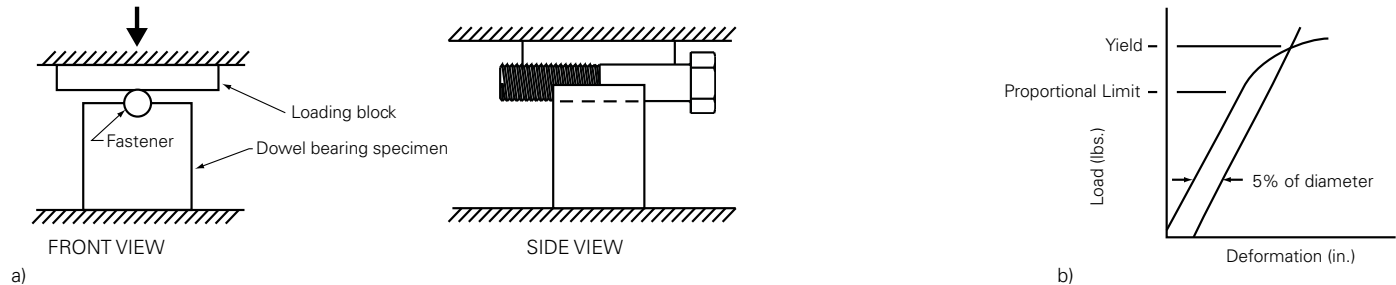
For wood, dowel bearing strengths are determined from dowel bearing tests in accordance with methods outlined in ASTM D5764 *Standard Test Method for Evaluating Dowel-Bearing Strength of Wood and Wood-Based Products* [2, 3]. These tests are used to define the specific gravity-based relationships for wood members in Table A1. Dowel bearing strengths for structural composite lumber are specific to each manufacturer's product and are typically reported on the basis of an equivalent specific gravity,  $G_{eqv}$ , using equivalency methods described in ASTM D5456 *Standard Specification for Evaluation of Structural Composite Lumber Products* [4]. The bearing strength and associated equivalent specific gravity may vary by product grade, fastener type, diameter, and orientation. The equivalent specific gravities are generally not equivalent or correlated to the actual product density.

For wood structural panels, a value of specific gravity,  $G$ , is provided that establishes the appropriate  $F_e$  for use in calculating lateral connection design values,  $Z$ , but generally does not represent the actual product density. For example, the specific gravity applicable for determination of  $F_e$  for wood structural panels, plywood and oriented strand board (OSB), with  $D \leq \frac{1}{4}$  inch is based on results of dowel bearing strength tests. A value of specific gravity is derived from testing to establish the appropriate bearing strength when used in the applicable bearing strength equation [5]. For plywood used in bolted joints, where fastener diameter is greater than  $\frac{1}{4}$  inch, the use of dowel bearing strength of 5600 psi regardless of plywood species and direction of face grain has been shown to result in conservative design values when compared to the tested strength of bolted plywood joints [6].

An illustration of a typical half-hole dowel bearing test specimen used to determine dowel bearing strength of a wood specimen is shown in Figure A1.

### Figure A1. Dowel Bearing Strength Test Specimen

a) Dowel bearing strength half-hole specimen, and b) illustration of 5% diameter offset yield limit state for dowel bearing strength.



For steel designed in accordance with AISC-360 *Specification for Structural Steel Buildings*, the nominal bearing strength is  $2.4F_u$  when deformation at the bolt hole is a design consideration [7]. The resulting nominal bearing stress is 139 ksi for ASTM A36 steel with  $F_u = 58$  ksi [8]. In the NDS, tabulated design values for wood-to-steel connections using these steel bearing stresses are divided by 1.6, providing a dowel bearing strength,  $F_e$ , of 87,000 psi. While the 1.6 factor is intended to permit use of load duration increases for calculation of the adjusted lateral design value,  $Z'$ , per the NDS, it is not intended to supersede requirements for design of metal parts in accordance with accepted engineering practice (see NDS 10.2.3).

For steel designed in accordance with AISI S100 *North American Specification for the Design of Cold-Formed Steel Structural Members*, the nominal bearing stress is determined by the expression  $(4.64t + 1.53)F_u$ , which is applicable where deformation around a bolt hole is a design consideration [9]. The resulting ASTM A653 [10] nominal bearing stress for steel with  $F_u = 45$  ksi ranges from 76 ksi to 119 ksi for thicknesses ranging from 0.036 to 0.239 inches, respectively. In the NDS, tabulated design values for wood-to-steel connections are based on  $2.2F_u$  divided by 1.6, providing a dowel bearing strength,  $F_e$ , of 61,850 psi. The use of  $2.2F_u$  for the range of steel thicknesses addressed in NDS wood-to-steel connection design value tables for ASTM A653 steel is based on bearing stress provisions in prior editions of AISI S100 in lieu of more recent AISI S100 provisions that adjust bearing stress based on steel thickness.

This bearing strength derivation approach for steel, which includes dividing the nominal bearing stress for

steel by 1.6 when calculating the lateral connection design value,  $Z$ , is intended to apply to steel of other thicknesses, with other specified minimum tensile strengths, and that vary from ASTM A36 and ASTM A653 steel, for which steel design standards are applicable.

For stainless steel [11,12] designed in accordance with AISC *Design Guide 27 Structural Stainless Steel* for hot-rolled structural stainless steel, the nominal bearing strength is limited to maximum value of  $1.25F_u$  when deformation at the bolt hole is a design consideration [13]. Dividing this bearing stress value by 1.6 for consistency with the bearing strength approach for carbon steel, provides a dowel bearing strength,  $F_e$ , of  $1.25F_u/1.6$ . For cold-formed structural stainless steel designed in accordance with SEI/ASCE 8-02 *Specification for the Design of Cold-Formed Stainless Steel Structural Members*, the nominal bearing strength for single shear connections with a washer under the bolt head and nut is  $2.0F_u$  [14]. Dividing this bearing stress value by 1.6 for consistency with the bearing strength approach for carbon steel, provides a dowel bearing strength,  $F_e$ , of  $2.0F_u/1.6$ .

For bolts bearing on aluminum designed in accordance with ADM-1 *Aluminum Design Manual*, the nominal bearing stress on bolts is  $2F_u$  [15]. Dividing this bearing stress value by 1.6 for consistency with the bearing strength approach for steel, provides a dowel bearing strength,  $F_e$ , of  $2F_u/1.6$ .

Theoretical modeling of dowel bearing strength of concrete and dowel bearing tests in concrete with maximum  $f'_c = 2700$  psi are the basis of the relationship  $3f'_c$  representing the approximate 5% offset limit state value for concrete dowel bearing strength [16, 17]. Information on design of dowels in concrete pavements suggests

dowel bearing strength of concrete in this range, and also dependence on dowel diameter with larger bearing strengths associated with dowels of smaller diameter [18]. For the design of wood-to-concrete connections, a concrete dowel bearing strength of 7500 psi is considered applicable for concrete with a minimum compressive strength of 2500 psi. The use of a dowel bearing strength of concrete for calculation of lateral connection design values,  $Z$ , is not intended to supersede requirements for

design of concrete or anchors in concrete in accordance with accepted practice [19] (see NDS 10.2.4).

NDS reference values of dowel bearing strength,  $F_e$ , in Table A1 and in NDS Table 12.3.3 are rounded values and are generally associated with smaller and larger values than would result from the direct use of the equations. The rounded values of  $F_e$  are used to calculate the lateral connection design values,  $Z$ , tabulated in the NDS.

**Table A1. Dowel Bearing Strength,  $F_e$ .**

Material	Nominal Bearing Stress	NDS Dowel Bearing Strength, $F_e$ , Equation (psi)	NDS Reference Value, $F_e$ (psi)
<b>Wood members<sup>3</sup> (for <math>D \geq 1/4''</math>)</b>			
Parallel to grain ( $F_{e\parallel}$ )		11200 G	NDS Table 12.3.3
Perpendicular to grain ( $F_{e\perp}$ )		$6100 G^{1.45}/D^{0.5}$	NDS Table 12.3.3
<b>Wood members<sup>3</sup> (<math>D &lt; 1/4''</math>)</b>			
Parallel and perpendicular to grain ( $F_e$ )		$16600 G^{1.84}$	NDS Table 12.3.3
<b>Wood Structural Panels</b>			
<b>Plywood (for <math>D \leq 1/4''</math>)</b>			
Structural 1, Marine ( $G = 0.5$ )		$16600 G^{1.84}$	4650
Other Grades ( $G = 0.42$ )		$16600 G^{1.84}$	3350
<small>(Note: Use <math>G = 0.42</math> when species of the plies is not known. When species of the plies is known, specific gravity listed for the actual species and the corresponding dowel bearing strength may be used, or the weighted average may be used for mixed species.)</small>			
<b>Plywood (for <math>D &gt; 1/4''</math>)</b>			
All Grades ( $G = 0.5$ )		11200 G	5600
<b>Oriented Strand Board (for <math>D \leq 1/4''</math>)</b>			
All Grades ( $G = 0.5$ )		$16600 G^{1.84}$	4650
<b>Concrete<sup>1</sup> (<math>f'_c = 2500</math> psi)</b>		$3f'_c$	7500
<b>Steel<sup>2</sup></b>			
ASTM A36 ( $F_u = 58$ ksi, $t > 1/4''$ )	$2.4F_u$	$2.4F_u/1.6$	87,000
ASTM A653 ( $F_u = 45$ ksi, $t < 0.239''$ )	$(4.64t + 1.53) F_u$	$2.2F_u/1.6$	61,850
<b>Stainless Steel<sup>2</sup></b>			
ASTM A240 (hot-rolled, $t \geq 1/8''$ )	$1.25F_u$	$1.25F_u/1.6$	
ASTM A240 (cold-formed)	$2.0 F_u$	$2.0 F_u/1.6$	
ASTM A240 specified $F_u$			
Type 304: $F_u = 75$ ksi			-
Type 304L: $F_u = 70$ ksi			-
Type 316: $F_u = 75$ ksi			-
Type 316L: $F_u = 70$ ksi			-
<b>Aluminum<sup>2</sup></b>		$2F_{tu}$	$2F_{tu}/1.6$

<sup>1</sup> Design of concrete parts shall also be in accordance with accepted practices (see NDS 10.2.4). For  $f'_c$  values greater than or equal to 2500 psi, the dowel bearing strength,  $F_e$ , is limited to 7500 psi.

<sup>2</sup> Design of metal parts shall also be in accordance with applicable metal design procedures (see NDS 10.2.3).

<sup>3</sup> For structural composite lumber, an equivalent specific gravity,  $G_{eqv}$ , is used to derive the dowel bearing strength instead of the actual product specific gravity,  $G$ .

## A.4 Dowel Bending Yield Strength

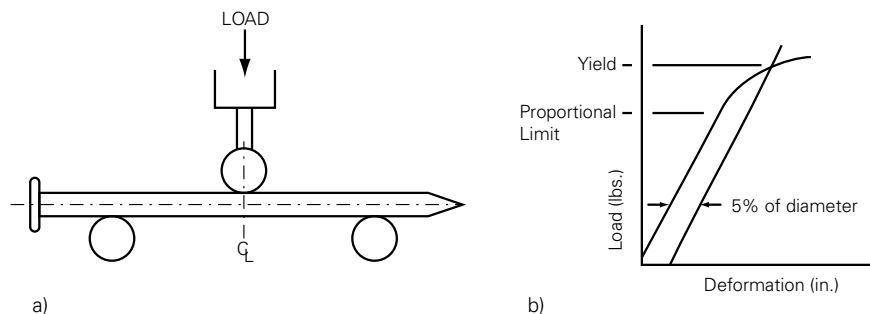
Dowel bending yield strengths,  $F_{yb}$ , for bolts, lag screws, wood screws, and nails for use in calculation of reference lateral design values,  $Z$ , are summarized in Table A2.

For nails and small-diameter wood screws and lags screws, the value of  $F_{yb}$  increases with reduced diameter and depends on the hardness of the steel. The empirical relationships are based on testing of carbon steel nails in accordance with procedures outlined in ASTM

F1575 *Standard Test Method for Determining Bending Yield Moment of Nails* for determination of the 5% offset limit state value [20, 21]. A typical fastener bending test specimen is shown in Figure A2. Stainless steel nails have been shown to display similar bending strength to comparably sized carbon steel fasteners [22]. For either carbon steel nails or stainless steel nails, bending yield values are evaluated in accordance with supplementary requirements of ASTM F1667 *Standard Specification for Driven Fasteners: Nails, Spikes, and Staples* for engineered construction [23].

### Figure A2. Dowel Bending Strength

a) Nail bending specimen, and b) illustration of 5% diameter offset yield limit state for bending yield strength.



For bolts and lag screws with diameter of  $\frac{3}{8}$  inch and greater,  $F_{yb} = 45$  ksi is associated with the equation  $F_{yb} = F_y/2 + F_u/2$  for bolts having  $F_y = 36$  ksi and  $F_u = 60$  ksi. This bending yield strength equation applies for steel fasteners for which steel design standards are applicable, including both carbon steel and stainless steel [24, 25, 26].

NDS reference values of fastener bending yield strength,  $F_{yb}$ , in Table A2 are rounded values. The rounded values of  $F_{yb}$  are used to calculate lateral connection design values,  $Z$ , tabulated in the NDS. In some cases, such as for nails of 0.344 to 0.375 inches in diameter as well as hardened nails greater than 0.192 inch, the equations produce slightly greater values of  $F_{yb}$  than used as the basis of lateral connection design values,  $Z$ , tabulated in the NDS.



**Table A2. Dowel Bending Yield Strength,  $F_{yb}$**

Fastener	NDS Dowel Bending Yield Strength, $F_{yb}$ , Equation (psi)	NDS Reference Value, $F_{yb}$ (psi)
Bolt, lag screw (with $D \geq \frac{3}{8}$ "), drift pin		
SAE J429 Grade 1: $F_y = 36$ ksi, $F_u = 60$ ksi	$F_y/2 + F_u/2$	45,000
ASTM A320, Class 1, Type B8 and B8M:		
Stainless Steel S30400: $F_y = 30$ ksi, $F_u = 75$ ksi		-
Stainless Steel S30403: $F_y = 25$ ksi, $F_u = 70$ ksi		-
Stainless Steel S31600: $F_y = 30$ ksi, $F_u = 75$ ksi		-
Stainless Steel S31603: $F_y = 25$ ksi, $F_u = 70$ ksi		-
Common, box, or sinker nail, spike, lag screw, wood screw (low to medium carbon steel)	130,400 – 213,900D	
0.099" $\leq D \leq 0.142$ "		100,000
0.142" $< D \leq 0.177$ "		90,000
0.177" $< D \leq 0.236$ "		80,000
0.236" $< D \leq 0.273$ "		70,000
0.273" $< D \leq 0.344$ "		60,000
0.344" $< D \leq 0.375$ "		45,000
Hardened steel nail (medium carbon steel) including post-frame ring shank nails	169,500 – 278,000D	
0.120" $\leq D \leq 0.142$ "		130,000
0.142" $< D \leq 0.192$ "		115,000
0.192" $< D \leq 0.207$ "		100,000

**A.5 References**

[1] ANSI/AWC NDS-2012 *National Design Specification (NDS) for Wood Construction*. American Wood Council, Leesburg, VA, 2012.

[2] ASTM D5764-97a (2013) *Standard Test Method for Evaluating Dowel-Bearing Strength of Wood and Wood-Based Products*. ASTM, West Conshohocken, PA, 2013.

[3] Wilkinson, T.L., *Dowel Bearing Strength*. Forest Products Laboratory Research Paper FPL-RP-505. Madison, WI: U.S. Department of Agriculture, Forest Service, Forest Products Laboratory, 1991.

[4] ASTM D5456-11a, *Standard Specification for Evaluation of Structural Composite Lumber Products*. ASTM, West Conshohocken, PA, 2011.

[5] APA D510, *Panel Design Specification*, APA – The Engineered Wood Association, Tacoma, Washington, 2012.

[6] APA E825E, *Fastener Loads for Plywood – Bolts*, APA – The Engineered Wood Association, Tacoma, Washington, 1997.

[7] AISC 360-10 *Specification for Structural Steel Buildings*. American Institute of Steel Construction, Inc., Chicago, IL, 2010.

[8] ASTM A36-12, *Standard Specification for Carbon Structural Steel*. ASTM, West Conshohocken, PA, 2012.

[9] AISI S100-2007. *North American Specification for the Design of Cold-Formed Steel Structural Members*. American Iron and Steel Institute, Washington, DC, 2007.

[10] ASTM A653-13, *Standard Specification for Steel Sheet, Zinc-Coated (Galvanized) or Zinc Iron Alloy-Coated (Galvannealed) by the Hot-Dip Process*. ASTM, West Conshohocken, PA, 2013.

[11] ASTM A240-14, *Standard Specification for Chromium and Chromium-Nickel Stainless Steel Plate, Sheet, and Strip for Pressure Vessels and for General Applications*. ASTM, West Conshohocken, PA, 2014.

- [12] ASTM A480-14b, *Standard Specification for General Requirements for Flat-Rolled Stainless and Heat-Resisting Steel Plate, Sheet, and Strip*. ASTM, West Conshohocken, PA, 2014.
- [13] AISC Design Guide 27, *Structural Stainless Steel*. American Institute of Steel Construction, Inc., Chicago, IL, 2013.
- [14] SEI/ASCE 8-02 *Specification for the Design of Cold-Formed Stainless Steel Structural Members*. American Society of Civil Engineers, Reston, VA, 2002.
- [15] AA ADM-1 2010. *Aluminum Design Manual, 2010 Edition*. Aluminum Association, Washington, D.C, 2010.
- [16] Vintzeleou, E.N., Tassios, T.P., "Mathematical Models for Dowel Action Under Monotonic and Cyclic Conditions." *Magazine of Concrete Research*: Vol. 38, No. 134: March 1986.
- [17] Biolzi, L., and Giuriani, E., "Bearing Capacity of a Bar Under Transversal Loads," *Materials and Structures*, V. 23, n 138, Nov., 1990, pp. 449 456, University di Udine, Udine, Italy, 1991.
- [18] Snyder, M.B., *Guide to Dowel Load Transfer Systems for Jointed Concrete Roadway Pavements*. National Concrete Consortium, Ames, IA, 2011.
- [19] ACI 318-14. *Building Code Requirements for Structural Concrete and Commentary*. American Concrete Institute, Farmington Hills, MI, 2014.
- [20] Loferski, J.R. and McLain, T.E., "Static and Impact Flexural Properties of Common Wire Nails," *Journal of Testing and Evaluation*, JTEVA, Vol. 19, No. 4, July 1991, pp. 297-304.
- [21] ASTM F1575-03 (2013) *Standard Test Method for Determining Bending Yield Moment of Nails*. ASTM, West Conshohocken, PA, 2013.
- [22] Rammer, D.R., and Zelinka, S.L., "Withdrawal Strength and Bending Yield Strength of Stainless Steel Nails," *Journal of Structural Engineering*, July 2014.
- [23] ASTM F1667-11a *Standard Specification for Driven Fasteners: Nails, Spikes, and Staples*. ASTM, West Conshohocken, PA, 2011.
- [24] SAE J429, *Mechanical and Material Requirements for Externally Threaded Fasteners*, Society of Automotive Engineers, Warrendale, PA, 1999.
- [25] ASTM A320-14, *Standard Specification for Alloy-Steel and Stainless Steel Bolting for Low-Temperature Service*. ASTM, West Conshohocken, PA, 2014.

# **American Wood Council**

## **AWC Mission Statement**

*To increase the use of wood by assuring the broad regulatory acceptance of wood products, developing design tools and guidelines for wood construction, and influencing the development of public policies affecting the use and manufacture of wood products.*

**American Wood Council**  
**222 Catocin Circle, SE**  
**Suite 201**  
**Leesburg, VA 20175**  
**[www.awc.org](http://www.awc.org)**  
**[info@awc.org](mailto:info@awc.org)**

